A B-spline Surface Stitching Algorithm Based on Point Cloud Data

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Xuedong Jing[1] **and Yuwei Zhang**[2]

Abstract. An algorithm to achieve smooth stitching between curved patches is presented. The algorithm adopts the inverse of the B-spline to find the control vertices of the common boundary of two curved blocks or multiple blocks; and then, the control vertex column vector that satisfies the condition given, which is determined by application of the continuous condition of surface G^1, is applied to substituting the original control vertex. In this algorithm, the smoothness of stitched surface is higher, and smooth stitching can be achieved by modifying one set of control vertices. The conditions for smooth stitching of two surfaces are verified, and the smooth stitching degree of the algorithm under different parameters is also tested.

Keywords: B-spline, surface stitching, geometric continuity.

1 Introduction

The geometric continuity condition between free-form surfaces is applied to constructing smooth surfaces on arbitrary topological regions. The article about the geometric continuity of B-spline surface is rare. More often, Bezier surface is used

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as the surface model. However, Bezier surface modeling has the disadvantages of poor fitting effect and more surface number. The B-spline surface can fit large surface patches with high precision and has its inherent smoothness. To achieve strict geometric continuous smooth B-spline surface modeling, we must first solve the problem of smooth splicing conditions between B-spline surfaces.

In 1990, W.H.Du[1] proposed the condition of G^1 splicing between two and several Bezier patches, but it did not involve the geometric continuous splicing condition of B-spline surface. In 2002, Xiquan Shi, Yan Zhao[2], two pairs of cubic B-splines The continuous condition of G^1 between surfaces is studied, and the necessary and sufficient conditions for satisfying G^1 continuity of two B-spline surfaces during splicing are given, but the B-spline surface is split into multiple Bezier surfaces and then found. The relationship between the vertices and the vertices is studied. Xiangyu Che and Xuezhang Liang[3] not only pointed out the essential difference between B-spline surface and Bezier surface in splicing, but also studied the continuous and sufficient conditions of G^1 satisfying NURBS surface. The algorithm is to splicing the B-spline patches and does not need to split. Based on the known necessary and sufficient conditions, the correction of the control vertices makes the splicing of the patches satisfy the smoothing condition.

2 B-spline Curve Surface Basis

2.1 B-spline Curve Surface Recursion Formula

The Bezier curve lacks flexibility in the application and is limited by the vertices. When the number of vertices of the feature polygon is determined, the order of the curve is determined, so the controllability is poor. When the number of vertices is large, the order of the curve will be higher. At this time, the control of the shape of the curve by the feature polygon will be significantly weakened. In addition, the Bezier curve defines that any point in the interval of $(0<t<1)$ is affected by the vertices, which makes it impossible to locally modify the curve, which greatly limits the possibility of actual modeling.

In order to better adapt to the needs of the actual modeling, the structural curve can be locally modified, closer to the feature polygon, lower order and easier to construct, and the B-spline curve comes into being. The mathematical expression of the B-spline curve is as follows:

$$P_{i,n}(t) = \sum_{k=0}^{n} P_{i+k} \cdot N_{k,n}(t), \quad (0 \le t \le 1) \tag{2.1}$$

Where P_{i+k} is the control vertex and $N_{k,n}(t)$ is the B-spline basis function, which is determined by the node vector T: $t_0 \le t_1 \le \cdots \le t_{n+k}$. It can be seen from (2.1) that the B-spline curve is segmentally defined by the control vertices. If m+n+1 vertices are given, the parameter curve of m+1 segments n times can be defined. The expression of $N_{k,n}(t)$ is as follows:

$$\begin{cases} N_{k,n}(t) = \dfrac{1}{n!} \displaystyle\sum_{j=0}^{n-k} (-1)^j \cdot C_{n+i}^{j} \cdot (t+n-k-j)^n \\ C_{n+i}^{j} = \dfrac{(n+i)!}{j!\,(n+i-j)!} \end{cases} \tag{2.2}$$

In formula, $0 \le t \le 1$, $k=0,1,2,\cdots,n$

When we read the i-th vertex, the polygonal polyline segment obtained by connecting the vertices with the line segments in turn is the characteristic polygon of the B-spline curve, and the vertices applied to the surface represent the control vertex mesh.

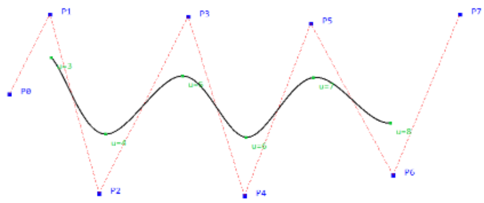

Figure 1 Cubic B-spline curve

Due to the nature of the segmentation representation, the cubic B-spline curve is defined by four adjacent control vertices whose expression is:

$$P(t) = N_{0,3}(t) \cdot P_0 + N_{1,3}(t) \cdot P_1 + N_{2,3}(t) \cdot P_2 + N_{3,3}(t) \cdot P_3 \tag{2.3}$$

It can be seen from equation (2.3) that a cubic B-spline curve defined by n vertices is connected by segmentation curves of n-3 segments, and the second-order continuous condition is satisfied at the joint.

The B-spline surface is a point column $P_{i,j}(i = 0,1,2, \dots, n), (j = 0,1,2, \dots, m)$ in space given (n+1)×(m+1) points; Connect the adjacent two points in the point sequence $P_{i,j}$ to construct a feature mesh, constructing a B-spline curve in the u and v directions to form a parametric polynomial surface in the form of tensor product, and set the node vector $U=\{u_i\}_{i=-\infty}^{+\infty}, V=\{v_j\}_{j=-\infty}^{+\infty}$ are the divisions of the u-axis and the v-axis of the parameter uv plane, respectively.
The B-spline surface definition is:

$$P(u, v) = \sum_{i=0}^{n}\sum_{j=0}^{m} P_{i,j}\, N_{i,k}(u) N_{j,h}(v) \tag{2.4}$$

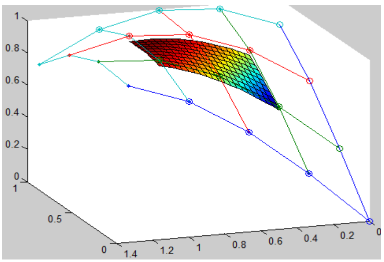

Figure 2 B-spline control grid surface

2.2 B-spline Curve Back Calculation

Given n+1 data points $p_i(i = 0,1, \dots, n)$, the general algorithm [4] is to use the first and last data points p_0 and p_n as the two endpoints of the B-spline interpolation curve, respectively. The remaining data points p_1, $p_2 \cdots$, p_{n-1} are sequentially used as the segment connection points of the B-spline interpolation curve. The node of the control point p_i is $\mathrm{u}_{i+k}(i = 0,1, \dots, m)$, the node vector $\mathrm{U}=[u_{0,} u_{1,} \dots, u_{n+k+1}]$. m+1 linear equations with n+1 control vertices as unknown vectors can be given by interpolation condition:

$$P(u_i) = \sum_{i=0}^{n} d_j \cdot N_{j,k}(u_i) = \sum_{j=i-k}^{i} d_j \cdot N_{j,k}(u_i) = q_{i-k}, u \in [u_i, u_{i+1}] \tag{2.5}$$

The node values in the curve definition domain $u \in [u_i, u_{i+1}]$ are sequentially substituted into the equation to satisfy the interpolation condition, namely:

$$\begin{cases} P(u_i) = \sum_{j=i-3}^{i} d_j \cdot N_{j,3}(u_i) = q_{i-3}, i = 3,4,\dots,n \\ P(u_{n+1}) = \sum_{j=n-3}^{n} d_j \cdot N_{j,3}(u_{n+3}) = q_m \end{cases} \tag{2.6}$$

2.3 Geometric Continuity Analysis of Curved Surfaces

For the curve, G^0 continuous means that the two curve segments have a common connection point, and G^1 continuous means that the two curve segments have the same unit tangent at the connection point, ie, the tangent vector, except that the G^0 continuity is satisfied[2]. The direction is the same or the tangential direction is continuous. G^2 continuous means that the two curved segments have the same curvature direction at the joint except that G^1 is continuous. Specific to the cubic B-spline curve, there is the following expression: two B-spline curves are provided

$$B(u) = \sum_{i=0}^{n} b_i \cdot N_{i,3}(u), C(v) = \sum_{j=0}^{m} c_j \cdot N_{j,3}(v) \tag{3.1}$$

G^0 continuous means that the two curved surfaces have a common boundary line; G^1 means that the two curved surfaces have continuous tangent planes on the common boundary line on the basis of G^0 continuous; G^2 is continuous in G^1 on the basis of the two surfaces, there is a continuous principal curvature on the common boundary line.

Set two cubic B-spline surfaces:

$$\begin{cases} B(u,v) = \sum_{i=0}^{m}\sum_{j=0}^{n} b_{ij} \cdot N_{i,3}(u)N_{j,3}(v) & (3.2) \\ C(s,v) = \sum_{i=0}^{g}\sum_{j=0}^{n} c_{ij} \cdot N_{i,3}(s)N_{j,3}(v) & (3.3) \end{cases}$$

The node vectors of the basis functions $N_{i,3}(u)$,$N_{i,3}(s)$,$N_{j,3}(v)$ are defined as:

$$\begin{cases} U = [0,0,0,0,u_3,\dots,u_m,1,1,1,1] \\ S = [0,0,0,0,s_3,\dots,s_g,1,1,1,1] \\ V = [0,0,0,0,v_3,\dots,v_g,1,1,1,1] \end{cases} \tag{3.4}$$

The conditions for continuity of the two surfaces are as follows:

$$\begin{cases} B(0,v) = C(0,v) \\ (\frac{\partial B}{\partial u},\frac{\partial C}{\partial s},\frac{\partial B}{\partial v})|_{B(0,v)=0} \end{cases} \tag{3.5}$$

3 Research on Bicubic B-spline Surface Stitching Algorithm

3.1 Two Pieces of Bicubic B-spline Surface G^1 Continuous Splicing

Although the predecessors have already demonstrated the smoothing conditions of the B-spline surface splicing in detail, there may be errors in the actual measurement of the point cloud data, so that the surface splicing of the originally smooth measured object does not necessarily satisfy the smoothing condition when generating the three-dimensional surface. Therefore, the algorithm is to achieve the correction of the local surface, which not only solves the problem that the complete data can be fitted in the three-dimensional scanning work, but also can correct the feature area, and is more suitable for the accurate measurement technology. The core of the algorithm is to use the back-calculation of B-spline to find the vertices of the relevant control points of the two surfaces, and on this basis, the two surfaces can be smoothly spliced.

There are two bicubic B-spline surfaces $B(u,v)$ and $C(s,v)$. Can the two bicubic B-spline patches known to be G^1 continuous splicing, which only has two common boundaries? A column of control vertices on the side is easy to implement and has good interactivity when applied to engineering problems. Assuming that they have a common boundary $\varphi(v)$, the surfaces $B(u,v)$ and $C(s,v)$ need to satisfy the condition of reaching G^1 continuously, Deduced by equation (2.6) as follows:

$$B(0,v) = c_0(v) \cdot C(0,v) \tag{3.6}$$

$$\alpha(v)\frac{\partial B}{\partial u}|_{u=0} + c_0(v)\cdot\beta(v)\frac{\partial C}{\partial s}|_{s=0} + c_0(v)\cdot\gamma(v)\frac{\partial C}{\partial v}|_{s=0} = 0 \quad (3.7)$$

Where $c_0(v)$、$\alpha(v)$、$\beta(v)$ 、 $\gamma(v)$ are functions of the common boundary parameters, and the geometric meaning is that there are common tangent planes at the common boundary of the two patches. In the calculation process, the parameters are polynomials. We can simplify $\alpha(v) = \alpha,\ \beta(v) = \beta$ as constant coefficients, and let $\gamma(v)$ be a piecewise linear function $\gamma(v) = \gamma_0(1-v) + \gamma_1(v)$, let the control vertex of the common boundary be H_j. Formula (3.7) into: (3.8)

$$\alpha\sum_{j=0}^{n}(b_{1j} - b_{0j})\cdot N_{j,3}(v) + \beta\sum_{j=0}^{n}\left(c_{1j} - c_{0j}\right)\cdot N_{j,3}(v) + \gamma(v)\varphi'(v) = 0$$

In fact $H_j = \{b_{0j}, c_{0j}\}$, what we have to do is to adjust α、β、$\gamma(v)$ and b_{1j}、c_{1j} to make the above formula. In equation (3.3), $\gamma(v)\varphi'(v)$ is the overall cubic polynomial curve. Due to the basis function $N_{j,3}(v)$,$\gamma(v)\varphi'(v)$ can be expressed in the following form:

$$\gamma(v)\varphi'(v) = \sum_{j=0}^{n} P_j\cdot N_{j,3}(v) \quad (3.9)$$

Among them.$P_0 = \gamma(0)\varphi'(0) = \gamma(0)\frac{3}{(v_4-v_3)}(H_1 - H_0)$ (3.10)

The interpolation properties of the B-spline curve are known:

$$\frac{3}{(v_4-v_3)}(P_1 - P_0) = [\gamma(1) - \gamma(0)]\frac{3}{(v_4-v_3)}(H_1 - H_0) \quad (3.11)$$

$$+\gamma(0)\frac{6}{(v_4-v_3)^2}(H_2 - 2H_1 + H_0)$$

Rephrase (3.8) to read:

$$\sum_{j=0}^{n}[\alpha(b_{1j} - b_{0j}) + \beta\left(c_{1j} - c_{0j}\right) + P_j]\cdot N_{j,3}(v) = 0 \quad (3.12)$$

If the base function is not equal to 0, then $\sum_{j=0}^{n}\alpha(b_{1j} - b_{0j}) + \beta\left(c_{1j} - c_{0j}\right) + P_j = 0$, the solution is $b_{1j} = b_{0j} - \frac{1}{\alpha}[\beta\left(c_{1j} - c_{0j}\right) + P_j]$. Replace the actual b_{1j} in the original $B(u,v)$ surface patch with the solved b_{1j}, so that the two curved slices $B(u,v)$ and $C(s,v)$ can achieve G^1 continuous stitching.

The specific algorithm is as follows:

1、 The common boundary curve $\varphi(v)$ is converted into an overall cubic B-spline curve. However, we still use $\varphi(v)$ to represent the adjusted common boundary curve. Then we use the B-spline curve reverse method to calculate $\varphi(v)$ as a cubic B-spline curve, which is recorded as:

$$\varphi(v) = \sum_{j=0}^{n} H_j \cdot N_{j,3}(v)$$

2、 Select the appropriate parameters, α、β、$\gamma(v)$

3、 Using the inverse of the B-spline method, find the cubic B-spline control vertex P_j of $\gamma(v)\varphi'(v)$.

4、 Find b_{1j} from the equation (3.12) and replace the actual control vertex b_{1j} in the original $B(u,v)$ surface patch.

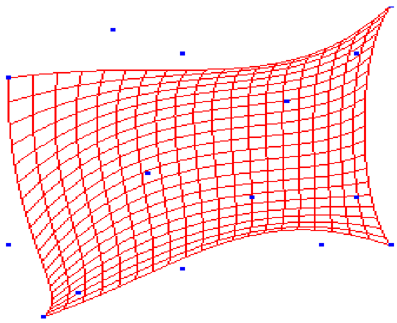

Figure 3 Splicing verification experiment

3.2 Three Pieces of Bicubic B-spline Surface G^1 Continuous Splicing

With three tensor plot B-spline surface:

$$\begin{cases} B(u,v) = \sum_{i=0}^{n_1}\sum_{j=0}^{n_2} b_{ij} \cdot N_{i,3}(u)N_{j,3}(v)\,, 0 \le u, v \le 1 & (3.13) \\ C(v,w) = \sum_{i=0}^{n_2}\sum_{j=0}^{n_3} c_{ij} \cdot N_{i,3}(v)N_{j,3}(w), 0 \le v, w \le 1 & (3.14) \\ D(w,u) = \sum_{i=0}^{n_3}\sum_{j=0}^{n_1} c_{ij} \cdot N_{i,3}(w)N_{j,3}(u), 0 \le u, w \le 1 & (3.15) \end{cases}$$

When we solved the algorithm of continuous splicing of two bicubic B-spline surfaces G^1, we continued to solve the splicing of multiple surfaces. In this problem, the problem of three-slice splicing is solved first. Obviously, the three surfaces only need to be stitched together in two or two. However, due to the contradiction between the three surfaces, the second control point (E) on the boundary curve and the closest point (G) to the off-angle point will affect whether the splicing of the surface satisfies the smoothing condition.

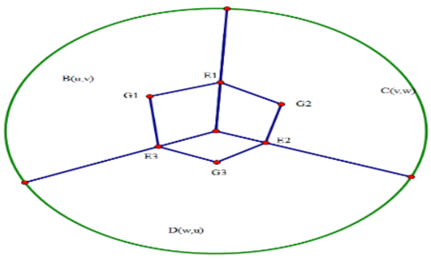

Figure 4 Stitching diagram

Let the three boundary curves be $\delta_1(u)\ \delta_2(v)\ \delta_3(w)$, and the algorithm based on the two surfaces is to find the corresponding α_i 、β_i、γ_i so that $\delta_1(u)\ \delta_2(v)\ \delta_3(w)$ is established. Let $\delta_1(u)\ \delta_2(v)\ \delta_3(w)$ control vertices, P_i、Q_i、R_i be $\gamma_1(v)\frac{\partial B}{\partial u}|_{v=0}\ \gamma_2(v)\frac{\partial C}{\partial u}|_{w=0}\ \gamma_3(v)\frac{\partial D}{\partial w}|_{u=0}$ control vertices. The purpose of our algorithm is not to construct three smooth-joined surfaces out of thin air, but to study how to modify the control vertices of the original surface so that they reach a smooth connection, with three surfaces already. Therefore, in our splicing process, we should try to keep the original control vertices smaller.

$$\begin{cases}\alpha_1(E_3-O)+\beta_1(E_2-O)+\dfrac{3\gamma_1}{u_4-u_3}(E_1-O)=0\\ \alpha_2(E_1-O)+\beta_2(E_3-O)+\dfrac{3\gamma_2}{v_4-v_3}(E_2-O)=0\\ \alpha_3(E_2-O)+\beta_3(E_1-O)+\dfrac{3\gamma_3}{w_4-w_3}(E_3-O)=0\end{cases} \tag{3.16}$$

The geometric meaning is that (E_1-O), (E_2-O), and (E_3-O) satisfy the coplanar condition, and the relevant parameters α_i 、β_i、γ_i are calculated. Actually, there is only one degree of freedom in α_i 、β_i、γ_i.

(3.17)

$$\begin{cases} \alpha_1(G_3 - E_1) + \beta_1(G_1 - E_1) + P_1 = 0 \\ \alpha_2(G_1 - E_2) + \beta_2(G_2 - E_2) + P_2 = 0 \\ \alpha_3(G_2 - E_3) + \beta_3(G_3 - E_3) + P_3 = 0 \end{cases}$$

To solve this problem, turn it into:

$$\begin{bmatrix} \beta_1 & 0 & \alpha_1 \\ \alpha_2 & \beta_2 & 0 \\ 0 & \alpha_3 & \beta_3 \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} \tag{3.18}$$

Among them

$$\begin{cases} W_1 = (\alpha_1 + \beta_1)E_1 - P_1 \\ W_2 = (\alpha_2 + \beta_2)E_2 - Q_1 \\ W_3 = (\alpha_3 + \beta_3)E_1 - R_1 \end{cases} \tag{3.19}$$

Where α_i、β_i can be calculated [5], the determinant of the coefficient matrix is not equal to 0, so the equation group has a unique solution, and the solution is G_1、G_2、G_3, replacing the original G_1、G_2、G_3.
The stitching algorithm for three B-spline surfaces is:

5、 The common boundaries of the surfaces B, C, and D are obtained by the inverse of the B-spline curve to obtain the control vertices H_i, I_i and J_i. The condition that needs to be satisfied is $H_0 = I_0 = J_0$, and $H_1 - H_0$、$I_1 - I_0$、$J_1 - J_0$ are coplanar.
6、 Given the parameter γ_i, α_i and β_i are solved by γ_i .
7、 Using the inverse B-spline curve method, the control vertices P_i, Q_i and R_i of $\gamma_1(v)\frac{\partial B}{\partial u}|_{v=0}$ $\gamma_2(v)\frac{\partial C}{\partial u}|_{w=0}$ $\gamma_3(v)\frac{\partial D}{\partial w}|_{u=0}$ are solved.
8、 Solved by the equations, the solution is G_1、G_2、G_3, replacing the original G_1、G_2、G_3.
9、 Two-piece splicing of three curved surfaces by using a splicing algorithm of two B-spline surfaces.

4 Algorithm, Experiment and Analysis

We test the feasibility of the algorithm by taking two and three surfaces as examples.
Firstly, the unit sphere is evenly divided into 24 pieces according to the longitude

and latitude, and the reverse processing of the B-spline surface is performed separately for each piece. The node vector is taken as: [0,0,0,0,0.25,0.5,0.75,1,1,1,1]. The lattice point is 5x5 dot matrix, and the obtained control points are 7x7 dot matrix. The two curved surfaces are judged by verifying the values of the three tangential determinants at the boundary of the two curved surfaces.

For the two surfaces, before the splicing, calculate the three-way tangential determinant at each point on the common boundary with a step of 0.001. The maximum value is 0.01596772, and the average value is 0.0095462.

Table 1. Algorithm 1 verifies test data.

Parameter value	maximum value	average value
$\alpha = 1.25, \beta = 1.25$ $\gamma(0) = 0.25, \gamma(1) = 1$	0.009653846	0.001362941
$\alpha = 1, \beta = 1$ $\gamma(0) = 1, \gamma(1) = 1$	0.094856112	0.018356527

It can be seen from the experiment that different parameter selection has a great influence on the smooth splicing of the surface. If the parameters are properly selected, the smoothness of the splicing is relatively good.

Acknowledgments

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