Fermatean Fuzzy Multiple Vehicle Routing Problem with Profits

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ABSTRACT: In real life while modeling and solving "Vehicle Routing Problem With Profits (VRPP)" we often face the state of uncertainty as well as hesitation due to various qualitative and quantitative factors. To deal with such situations, in this paper, we propose "Fermatean Fuzzy Multiple Vehicle Routing Problem With Profits (FFMVRPP)" in which we have considered the profits, expenses as Fermatean Fuzzy (FF) values and has included qualitative factors like 'Relation of customer with the seller', 'risk of transporting from one node to another' to make our model more realistic. The paper includes mathematical formulation of the FFMVRPP and proposed algorithms to solve them. We propose Fermatean Fuzzy Clarke and Wright's saving algorithm(FFCWA), Fermatean Fuzzy Nearest Neighbourhood algorithm and Nearest Neibourhood + Brute Force algorithm (Group First and brute second). An example is also stated to elaborate the given methods and ultimately we will conclude with the advantageous of a FFMVRPP over the classical VVRP along with the references.

KEYWORDS: Fermatean Fuzzy Sets, Vehicle Routing Problem with Profits, Fermatean Fuzzy Multiple Vehicle Routing Problem With Profits, Clarke and Wright's saving algorithm, Nearest Neighbourhood algorithm, Nearest Neighbourhood + Brute Force algorithm.

1 INTRODUCTION:

Vehicle Routing Problem was introduced by Dantzig and Ramser [3] approximately 60 years before, in the year 1959 and Clark and Wright proposed [6] an efective greedy heuristic for the approximate solution of the VRP. Classical Vehicle Routing Problems with Profits (VRPP) is an extension of the Vehicle Routing problem (VRP). The most basic problems of this class with only one route are often presented as variants of the Traveling Salesman Problem

(TSP) (Fischetti, Salazar González, and Toth) **[11]**. Though having a good practical interest in considering the profits in VRP, most of the existing literature revolves around the single-vehicle case of the problems. Among all the MVRPP, the only problem that has been studied in depth is the TOP(Team Orienteering problem) by "BUTT and CAVALIER" **[10]**.

Many authors have used fuzzy and intuitionistic fuzzy set theory for solving real life optimization problems such as transporting, routing etc **[4], [5], [7]**. Though having superiority over fuzzy, intutionistic and Phythagorean Sets **[1]**, not much work has been done with the Fermatean Fuzzy theory. This paper involves the application on FFS in VRPP.

Fuzzy theory **[2]** was introduces by zadeh (1965) . The use of fuzzy set theory became very rapid in the field of optimization after the pioneering work done by Bellman and Zadeh (1970) **[12]**.

Senapati and Yager introduced the concept of Fermatean fuzzy set (FFS)**[1]** which is one of the important generalizations of fuzzy set theory. The major advantage of FFS over fuzzy set is that FFS separates the degree of membership (acceptance level) and the degree of non-membership (non-acceptance level) of an element in the set. Later in 2020 some new important operations on Fermatean Fuzzy Sets by again given by Senapati and Yager **[9]**.

In this paper, we proposed FFMVRPP. The problem set up is as follow: Given a set of N total customers at any time t. The distributor has only a capacity of serving M customers out of N in a given unit of time, where $M \leq N$. The problem is to select M customers out of N and visit these M customers through different vehicle routes with the limitation that not more then K customers are visited in a single trip, where $K \leq M \leq N$. The objective here is to maximize the difference between the profit expectations level and serving expense level.

Corresponding to each customer a Fermatean Fuzzy profit is attached with its Fermatean Fuzzy degrees indicating the level of profit satisfaction offered by the customer. The relation of the customer with the distributor being an important qualitative feature, is again given by Fermatean fuzzy values. The risk of transporting goods from one node to another and cost matrix are given in Fermatean fuzzy terms which gives us the overall expense matrix.

The paper is structured in the following manner, in section 2 basic preliminaries and concepts of Fermatean fuzzy set theory along with its operations and vital functions have been reviewed. In section 3 a mathematical model of FFMVRPP has been proposed. Section 4 deals with the two step solution process of the FFMVRPP by the proposed algorithms - Fermatean Fuzzy

Clarke and Wrights savings algorithm, Fermatean Fuzzy Nearest Neighbourhood algorithm, Nearest Neighbourhood + Brute Force algorithm in sections 4.1, 4.2, 4.3, *and* 4.4 respectively. In section 5 an example has been presented with 7 customers and a depot node (O) out of which only 6 can be served in a unit time, where all the data is given in Fermatean Fuzzy environment. Section 6 contains a brief comparison of the results of the example. The last Section 7 comprises the concluding remarks and future works along with the references.

2 PRELIMINARIES AND CONCEPTS:[9],[1]

2.1 FERMATEAN FUZZY SETS (FFS)

Let X be universe of discourse. A Fermatean fuzzy sets F in X is an object having the form

$$F = \{< x,\ \alpha_F(x),\ \beta_F(x) > \ ; x\ \in X\}$$

Where,

$$\alpha_F(x) : X \to [\,0,1]\ and$$
$$\beta_F(x) : X \to [\,0,1]$$

including the condition

$$0 \leqslant (\alpha_F(x))^3 + (\beta_F(x))^3 \leqslant 1,\ \forall x \in X$$

The number $\alpha_F(x)$ and $\beta_F(x)$ denote respectively the degree of membership and the degree of non membership of the element x in the set F.
Note that , for the interest of simplicity, we shall mention the symbol

$$F = (\alpha_F(x), \beta_F(x))$$

for the Fermatean Fuzzy Sets,

$$F = \{< x, \alpha_F(x), \beta_F(x) >; x \in X\}$$

2.2 SCORE FUNCTION :

Let $F = (\alpha_F, \beta_F)$ be a FFS then the score function of F can be represented as

$$score(F) = \alpha_F^3 - \beta_F^3$$

For any FFS, $F = (\alpha_F, \beta_F)$ the suggested score function,
$score(F) \in [\,-1,1]$

2.3 ACCURACY FUNCTION:

Let $F = (\alpha_F, \beta_F)$ be an FFS, then the accuracy function regarding F can be described as follows:

$$acc(F) = \alpha_F^3 + \beta_F^3$$

Clearly, $acc(F) \in [\,0,1]$
Bigger the value of $acc(F)$, higher the accuracy of FFS, F will be.

2.4 DEGREE OF INDETERMINACY

For any FFS, F and $x \in X$

$$\pi_F(x) \;=\; \sqrt[3]{1-(\alpha_F(x))^3-(\beta_F(x))^3}$$

is identified as the degree of indeterminacy of x to F.

2.5 RANKING PRINCIPLE USING SCORE AND ACCURACY FUNCTION:

Let $F_1 = (\alpha_{F_1}, \beta_{F_1})$ and $F_2 = (\alpha_{F_2}, \beta_{F_2})$ be two $FFSs$. $score(F_i)$ and $acc(F_i)$, $i = 1,2$ are the score values and accuracy values of F_1 and F_2 respectively then the ranking principle using score and accuracy is defined as follows:

1. $If score(F_1) < score(F_2), then\ F_1 < F_2$
2. $If score(F_1) > score(F_2), then\ F_1 > F_2$
3. $If score(F_1) = score(F_2), then$
   (a) $If acc(F_1) < acc(F_2), then\ F_1 < F_2$
   (b) $If acc(F_1) > acc(F_2), then\ F_1 > F_2$
   (c) $If acc(F_1) = acc(F_2), then\ F_1 = F_2$

2.6 OPERATIONS ON FERMETEAN FUZZY NUMBERS:

Let $F = (\alpha_F(x), \beta_F(x))$, $F_1 = (\alpha_{F_1}(x), \beta_{F_1}(x))$ $F_2 = (\alpha_{F_2}(x), \beta_{F_2}(x))$ be three $FFSs$ and $\lambda > 0$, then their operations are defined as follows:

- $F_1 \cap F_2 = (min\{\alpha_{F_1}, \alpha_{F_2}\}, max\{\beta_{F_1}, \beta_{F_2}\})$

- $F_1 \cup F_2 = (max\{\alpha_{F_1}, \alpha_{F_2}\}, min\{\beta_{F_1}, \beta_{F_2}\})$

- $F_1 \boxplus F_2 = (\sqrt[3]{\alpha_{F_1}^3 + \alpha_{F_2}^3 - \alpha_{F_1}^3 \alpha_{F_2}^3}, \beta_{F_1} \beta_{F_2})$

- $F_1 \boxtimes F_2 = (\alpha_{F_1} \alpha_{F_2}, \sqrt[3]{\beta_{F_1}^3 + \beta_{F_2}^3 - \beta_{F_1}^3 \beta_{F_2}^3})$

- $F_1 \boxminus F_2 = (\sqrt[3]{\frac{\alpha_{F_1}^3 - \alpha_{F_2}^3}{1-\alpha_{F_2}^3}}, \frac{\beta_{F_1}}{\beta_{F_2}}) \ if \ \alpha_{F_1} \ \geq \ \alpha_{F_2}, \beta_{F_1} \ \leq \ min(\beta_{F_2}, \frac{\beta_{F_2} \pi_1}{\pi_2})$

- $F_1 \tilde{\div} F_2 = (\frac{\alpha_{F_1}}{\alpha_{F_2}}, \sqrt[3]{\frac{\beta_{F_1}^3 - \beta_{F_2}^3}{1-\beta_{F_2}^3}}) \ if \ \alpha_{F_1} \ \leq \ min(\alpha_{F_2}, \frac{\alpha_{F_2} \pi_1}{\pi_2}), \beta_{F_1} \ \geq \ \beta_{F_2}$

- $F^c = (\beta_F, \alpha_F)$

- $\lambda F = (\sqrt[3]{1 - (1 - \alpha_F^3)}, \beta_F^{\lambda})$

- $F^{\lambda} = (\alpha_F^{\lambda}, \sqrt[3]{1 - (1 - \beta_F^3)^{\lambda}})$

3 Mathematical Model:

Let $G = (V, A)$ be a complete graph with $V = \{O, 1, 2, ..., n\}$ is the of all nodes and A be the set of all arcs. The nodes $V \backslash \{O\} = \{1, 2, ..., n\}$ shows the n customers and the node O represents the **depot** from where any route starts or ends. For any $(i, j) \in A$, $\tilde{C}_{ij}$ denots the expense of travelling from one node to another, where the expense is taken as a weight function of travelling cost and risk, and a profit $\tilde{P}_i$ is associated with each customer $\forall i = 1, 2, ..n$. The profit of each sellected customer can be collected only once.
Out of the total N customers only M can be served/visited in a unit time and at most K customers can be visited in a single trip.
$\therefore$ Number of Vehicles or trips required (either we can assume to have multiple vehicles or to have one vehicles with multiple trips) to visit these selected M customers will be $\lfloor M \div K \rfloor = |R|$ (let).
The problem is to first select these M customers out of N and then to find

suitable routes so that the difference of profit satisfaction level and serving expense level is *maximized*.

3.1 NOTATIONS

N: Total number of available customer.
M: Number of customers to be served in a unit time with $M \leq N$
K: Maximum number of customers that can be visited in a single trip.
R: Set of all routes through which customers are visited.

$\therefore \mid R \mid = \lfloor M \div K \rfloor$

$\tilde{C}_{ij}$: Expense of travelling from i^{th} node to j^{th} node.
$\tilde{P}_i$: Profit satisfaction level offererd by the i^{th} customer.
$\tilde{W}_i$: Willingess to serve customer i or relation of i^{th} customer with seller
Let us define the following variables :
$y_{ir} = 1$ if $i \in V$ is visited by vehicle route $r \in R$ or 0 otherwise.
$x_{ijr} = 1$ if $(i, j) \in A$ is traversed by the vehicle route $r \in R$ or 0 otherwise.

$\therefore$ **The mathematical formulation is as follows:**

$$Max \ \{\alpha \tilde{\sum_{i \in V}} \tilde{p}_i \sum_{r \in R} Y_{ir} \boxminus \tilde{\sum_{(i,j) \in A}} \tilde{C}_{ij} \sum_{r \in R} X_{ijr}\}$$

Subject to

$$\sum_{j \in V} x_{ijr} = y_{ir}; \quad \forall i \in V, r \in R ....(1)$$

$$\sum_{j \in V} x_{jir} = y_{ir}; \quad \forall i \in V, r \in R ....(2)$$

$$\sum_{r \in R} y_{or} \ \leq \ \mid R \mid = [\frac{m}{k}] ....(3)$$

$$\sum_{r \in R} y_{ir} \leq 1; \quad \forall i \in V \backslash \{0\} ....(4)$$

$$\sum_{i} y_{ir} \leq k; \quad \forall i = 1, 2, ..., m \ \ \forall r \in R ....(5)$$

$$y_{ir} \in \{0, 1\}; \quad \forall i \in V, \ r \in R ....(6)$$

$$x_{ir} \in \{0, 1\}; \quad \forall (i, j) \in A, \ r \in R ....(7)$$

Here, the objective function maximizes the difference between the overall gained profit satisfaction level and the total expense level of serving. Constraints (1) and (2) ensures that one arc enters and one arc leaves for each visited vertax. Constraint (3) limits the number of routes to be at most $|R|$, while constraint (4) imposes that each customer is visited at most once. The constraint (5) for any route $r \in R$, at most K customers are visited. Finally (6) and (7) shows that y_{ir} and x_{ijr} can take only two values either 0 or 1, $\forall i \in V$ *and* $r \in R$.

4 Solution Methods

Clearly, FFMVRPP is a two stage problem. First we have to select M customers out of N then to create vehicle routes keeping in mind the constraints.
So we propose a two step solution procedure in which the first step deals with the sellection of M customers out of N and the second steps create efficient routes to visit the sellected customers from step 1.

4.1 Step 1 Procedure

For each customer we calculate their **attractiveness or favourability** depending on Profit satisfaction $\tilde{P}_i$, willingness or relation with seller $\tilde{W}_i$ and the expected expense in serving them $\tilde{C}_{ij}$. For this we use the following function : **Favourability Function**

$F(\alpha_1\tilde{P}_i,\ \alpha_2\tilde{W}_i,\ \alpha_3\tilde{C}_{0i}) = a_i^3 - a_i'^3 + b_i'^3 - b_i^3$ ....(8)
where $\alpha_1, \alpha_2, \alpha_3$ are weights and $\alpha_1 + \alpha_2 + \alpha_3 = 1$
and $\alpha_1\tilde{P}_i \boxplus \alpha_2\tilde{W}_i = (a_i, b_i)$ and $\alpha_3\tilde{C}_{0i} = (a_i', b_i')$
$\forall i = 1, 2, ...n$
equation (8) gives us the attractiveness of the customer i. Let F be the attractiveness of each customer and let we make a list F_i $\forall\ i = 1, 2, ...n$ and arrange the list in Descending order
Take a list L = customer i in F_i and select first m customers from the list $[F_i]$. Thus length of list L=M
$\therefore$ L is the list of selected customers.
$\therefore$ expected profit satisfaction $\widetilde{=}\ \sum_{i \in L} P_i \ \widetilde{=}\ \tilde{P}$ (let)
This concludes our STEP 1 of the solution procedure.

4.2 Fermatean Fuzzy C.W.A For Step 2

The algorithm proceeds as follows:

Calculating Savings
Let

$$\tilde{C_{oi}} \boxplus \tilde{C_{oj}} = (\alpha_{oj}, \beta_{oj})$$

$$\tilde{C_{ij}} = (c_{ij}, d_{ij})$$

$$\tilde{S_{ij}} = \alpha_{oi}^3 - C_{ij}^3 + d_{ij}^3 - \beta_{oj}^3$$

Create m individual routes (O, i, O) ; $i = 1, 2, ..m$
Create List $l = S_{ij}$ where $S_{ij} =$ defuzzify $\tilde{S}_{ij}$

(Computing savings for each merge of locations i and j. Making m routes $0-i-0$. List l keeps savings for each possible merge S_{ij})

Creating Routes/trips

Choose $max\ S_{ij},\ 1 \leq i \leq M$ and $i+1 \leq j \leq M$
let it is = $S_{i'j'}$
route r_i : Join i' and j' i.e.create $O-i'-j'-O$ and expense is
$\tilde{C}_i = \tilde{C}_{0i} \boxplus \tilde{C}_{ij} \boxplus \tilde{C}_{oj}$
eliminate $S_{i'j'}$ from list l and update list l

Any route can visit at most K customers
If length $r_i < K$ Then
choose $max\ S_{i'j}\ or\ S_{ij'}$ for $1 \leq i \leq M$; $i+1 \leq j \leq M$
without loss of generelity, let we take it as $S_{j'i''}$
extend r_i as $O-i'-j'-i''-O$ and thus
$\tilde{C}_i = \tilde{C}_{0i} \boxplus \tilde{C}_{i'j'} \boxplus \tilde{C}_{j'i''} \boxplus \tilde{C}_{i''O}$
eliminate $S_{j'i''}$ from l and update l.
else: end route r_i and initiate route r_{i+1} with updated l

Multiple Routes

Repeat step 4.3.3 , 4.3.4 untill Length $(l) = 0$
after this step we will have $|R|$ = $\lfloor M \div K \rfloor$ routes, each containing different set of customers and

$$\tilde{C} \doteqdot \sum_{i=1}^{|R|} \tilde{C}_i$$

Final Result

Ans = $\alpha\ \tilde{P} \boxminus \tilde{C}$
where $\tilde{C}$ is the overall expense level and $\tilde{P}$ is profit satisfaction level multiplied by a scalar $\alpha > 0$ indicating the satisfaction level of the whole process right from selection of customer to creating multiple routes and serving them in route order trips.

4.3 Fermatean Nearest Neighbourhood For Step 2

For step 2 i.e creating routes and trips the method is given as follows.

Initiating Route With Least Expensive Node

Take $\tilde{C}_{ij};\ \ O \leq i \leq M\ ,\ O \leq j \leq M.$
min $\tilde{C}_{Oj};\ 1 \leq j \leq M = \tilde{C}_{Oj_1}$ (let).
Initiate route $O-j_1$; where $j_1 \in 1,2,...,M$
expense $\tilde{C}_i = \tilde{C}_{Oj_1}$; Now update $\tilde{C}_{i,j}$ by eliminating $\tilde{C}_{Oj_1}$ and $\tilde{C}_{j_1O}$ i.e
$\tilde{C}_{i,j} \setminus \{\ \tilde{C}_{Oj_1}, \tilde{C}_{j_1O}\}$

Extend And Add Nodes

take $i = j_1$
$\implies$ min $\{\tilde{C}_{ij}\ ;\ 1 \leq j \leq M\ ;\ \text{i} \neq j\}\ =\ \tilde{C}_{ij_2}\ \ (let)$

$\Longrightarrow \tilde{C}_{ij_2} = \tilde{C}_{j_1 j_2}$; extend route O-j$_1$-$j_2$;
$\tilde{C}_i = \tilde{C}_{Oj_1} \boxplus \tilde{C}_{j_1 j_2}$
$update\ \tilde{C}_{i,j}$ matrix $\tilde{C}_{i,j} \backslash \{\tilde{C}_{j_1,j_2,j_2,j_1}\}$

Create Routes

Repeat step 5.2 q number of times where
$q = min\ \{M, K-1\}$ to get a route visiting q customers and ending with O.

$$route \quad r_i: \quad O - j_1 - j_2 - j_3 - .... - j_q - O$$

$$\tilde{C}_i = \tilde{C}_{Oj_1} \boxplus \tilde{C}_{j_1 j_2} \boxplus .... \boxplus \tilde{C}_{j_{q-2} j_{q-1}} \boxplus \tilde{C}_{j_q O}$$

$$update\ \tilde{C}_{ij}\ by\ \tilde{C}_{ij} \backslash\ \{\tilde{C}_{ij};\ i = j_1, j_2, ...j_q\ and\ j = j_1, j_2, ...j_q\}$$

Repeat steps 5.1, 5.2, 5.3 $[\frac{m}{k}] = R$ times to set $\mid R \mid$ different routes with update $\tilde{C_{ij}}$ and $m \in m - q$ customers.

$$\therefore\ Total\ expense \tilde{=} \sum_{i=1}^{R} \tilde{C}_i \tilde{=} \tilde{C}$$

Final Result

Final Result $= \alpha \tilde{p} \boxminus \tilde{C}$
where $\tilde{C}$ is the overall expense level and $\tilde{P}$ is profit satisfaction level multiplied by a scalar $\alpha > 0$ indicating the satisfaction level of the whole process right from selection of customer to creating multiple routes and serving them in route order trips.

4.4 Nearest Neighbourhood And Brute Force For Step 2

For the step 2 i.e creating routes we proceed as follows:

Grouping Customers

Group the customers using the above Nearest Neighbourhood Method *i.e.* section 5.1, 5.2, 5.3 and 5.4 except calculating $\tilde{C}_i$. A total of $\mid R \mid = [\frac{m}{k}]$ groups are to be made.

Case(i): **if** $k \mid m$ **then** We have $\mid R \mid$ groups, each containing exactly k customers.

Case(ii): **if** $k \nmid m$ **then**
We have $\mid R \mid -1$ groups of k customers and 1 group of $m - rk$ customers.

When K | M

For case(i) : For any group G_i, $i = 1, 2, 3, ..., \mid R \mid$ we have $k!$ number of different possible routes starting and ending with nodes O .
take

$$\tilde{r}_i = Min\{\tilde{r_{ij}}\ ;\ 1 \leq j \leq k!\}$$

take "$\tilde{r}_j$ expens" be the list of expenses of all $k!$ possible routes in group G_i , where,
$i = 1, 2, 3, ..., \mid R \mid$ and $j = 1, 2, 3, ..., k!$

take $\tilde{C_i} = Min\{\tilde{r}_j \ ; \ 1 \leq j \leq k!\}$

$$\therefore \ Total \ expense \ = \ \sum_{i=1}^{|R|} \tilde{C_i}$$

When K ∤ M

For case(ii) : Repeat step 6.2.2 for $|R|-1$ groups having k customers each.
For the group with $m-rk$ customers.
Number of different possible routes $= (m-rk)!$
Let $R_j^{'}$ be the expense of each possible route.

taking, $\tilde{C'} = Min \ \{ \ \tilde{R_j}^{'} \ | \ 1 \leq j \leq (m-rk)! \ \}$

$$\therefore \ TotalExpense \ = \ \sum_{i=1}^{|R|-1} \tilde{C_i} \boxplus \tilde{C'} \ \tilde{=} \ \tilde{C}$$

Final Result

Ans $= \ \alpha \tilde{P} \ - \ \tilde{C}$
where $\tilde{C}$ is the overall expense level and $\tilde{P}$ is profit satisfaction level multiplied by a scalar $\alpha > 0$ indicating the satisfaction level of the whole process right from selection of customer to creating multiple routes and serving them in route order trips.

5 Numerical Example

5.1 PROBLEM FORMULATION

Consider the following FFMVRPP with 7 customers and 1 depot O. The data are given in the following tables. All the data i.e profits, cost, willingness , risk are taken in FF terms. The problem is to First select 6 customers, as it is assumed that in 1 day only 6 customers can be visited, and then create routes to visit/serve these 6 customers assuming that in one trip only 3 customers can be visited. Here N=7 ; M=6 ; K=3.
Thus the no paths R= $\lfloor 6 \div 3 \rfloor = 2$

	$Customer_1$	$Customer_2$	$Customer_3$	$Customer_4$	$Customer_5$	$Customer_6$	$Customer_7$
Profit Satisfaction level	$\tilde{41}_{(0.86,0.36)}$	$\tilde{29}_{(0.52,0.85)}$	$\tilde{45}_{(0.97,0.08)}$	$\tilde{30}_{(0.55,0.83)}$	$\tilde{43}_{(0.91,0.24)}$	$\tilde{38}_{(0.78,0.54)}$	$\tilde{44}_{(0.94,0.16)}$
Willingness	$(0.66, 0.71)$	$(0.3, 0.97)$	$(0.77, 0.54)$	$(0.8, 0.4)$	$(0.91, 0.24)$	$(0.55, 0.83)$	$(0.94, 0.16)$
Expected Fuzzy Cost	$\tilde{8}_{(0.8,0.48)}$	$\tilde{10}_{(0.97,0.08)}$	$\tilde{2}_{(0.2,0.95)}$	$\tilde{1}_{(0.1,0.99)}$	$\tilde{9}_{(0.9,0.27)}$	$\tilde{1}_{(0.1,0.99)}$	$\tilde{9}_{(0.9,0.27)}$

Table 1: Selection Table

Table 1 give us the data of the profit level offered by all the customers, the willingness to serve them i.e. the data indicating the relation of the customers with the distribution and the expected fuzzy cost indicating that how much costly it is to serve/visit the customers directly.

	$\tilde{8}_{(0.8,0.48)}$	$\tilde{10}_{(0.97,0.08)}$	$\tilde{2}_{(0.2,0.95)}$	$\tilde{1}_{(0.1,0.99)}$	$\tilde{9}_{(0.9,0.27)}$	$\tilde{2}_{(0.1,0.99)}$	$\tilde{9}_{(0.9,0.27)}$
$\tilde{8}_{(0.8,0.48)}$		$\tilde{10}_{(0.97,0.08)}$	$\tilde{9}_{(0.9,0.27)}$	$\tilde{4}_{(0.4,0.93)}$	$\tilde{1}_{(0.1,0.99)}$	$\tilde{8}_{(0.8,0.48)}$	$\tilde{2}_{(0.2,0.95)}$
$\tilde{10}_{(0.97,0.08)}$	$\tilde{10}_{(0.97,0.08)}$		$\tilde{4}_{(0.4,0.93)}$	$\tilde{9}_{(0.9,0.27)}$	$\tilde{9}_{(0.9,0.27)}$	$\tilde{2}_{(0.2,0.95)}$	$\tilde{5}_{(0.5,0.87)}$
$\tilde{2}_{(0.2,0.95)}$	$\tilde{9}_{(0.9,0.27)}$	$\tilde{4}_{(0.4,0.93)}$		$\tilde{9}_{(0.9,0.27)}$	$\tilde{10}_{(0.97,0.08)}$	$\tilde{7}_{(0.7,0.65)}$	$\tilde{9}_{(0.9,0.27)}$
$\tilde{1}_{(0.1,0.99)}$	$\tilde{4}_{(0.4,0.93)}$	$\tilde{9}_{(0.9,0.27)}$	$\tilde{9}_{(0.9,0.27)}$		$\tilde{8}_{(0.8,0.48)}$	$\tilde{6}_{(0.6,0.78)}$	$\tilde{5}_{(0.5,0.87)}$
$\tilde{9}_{(0.9,0.27)}$	$\tilde{1}_{(0.1,0.99)}$	$\tilde{9}_{(0.9,0.27)}$	$\tilde{10}_{(0.97,0.08)}$	$\tilde{8}_{(0.8,0.48)}$		$\tilde{9}_{(0.9,0.27)}$	$\tilde{3}_{(0.3,0.97)}$
$\tilde{1}_{(0.1,0.99)}$	$\tilde{8}_{(0.8,0.48)}$	$\tilde{2}_{(0.2,0.95)}$	$\tilde{7}_{(0.7,0.65)}$	$\tilde{6}_{(0.6,0.78)}$	$\tilde{9}_{(0.9,0.27)}$		$\tilde{2}_{(0.2,0.95)}$
$\tilde{9}_{(0.9,0.27)}$	$\tilde{2}_{(0.2,0.95)}$	$\tilde{5}_{(0.5,0.87)}$	$\tilde{9}_{(0.9,0.27)}$	$\tilde{5}_{(0.5,0.87)}$	$\tilde{3}_{(0.3,0.97)}$	$\tilde{2}_{(0.2,0.95)}$	

Table 2: Fuzzy Cost Table

Table 2 give us the fuzzy cost of moving from one node to another. Here the matrix is symmetric matrix .i.e. the fuzzy cost for going from i^{th} to j^{th} and j^{th} to i^{th} node is same. All the diagonal cells are empty as the transportation is not required from i^{th} to i^{th} node. The fermatean fuzzy values indicates how much costly it is to move from the i^{th} to j^{th} node.

	$(0.41,0.84)$	$(0.78,0.6)$	$(0.27,0.9)$	$(0.83,0.55)$	$(0.65,0.7)$	$(0.52,0.78)$	$(0.92,0.43)$
$(0.41,0.84)$		$(0.83,0.55)$	$(0.65,0.7)$	$(0.88,0.49)$	$(0.27,0.9)$	$(0.41,0.84)$	$(0.65,0.7)$
$(0.78,0.6)$	$(0.83,0.55)$		$(0.78,0.6)$	$(0.41,0.84)$	$(0.78,0.6)$	$(0.94,0.37)$	$(0.52,0.78)$
$(0.27,0.9)$	$(0.65,0.7)$	$(0.78,0.6)$		$(0.83,0.55)$	$(0.78,0.6)$	$(0.71,0.66)$	$(0.27,0.9)$
$(0.83,0.55)$	$(0.88,0.49)$	$(0.41,0.84)$	$(0.83,0.55)$		$(0.43,0.92)$	$(0.94,0.37)$	$(0.11,0.96)$
$(0.65,0.7)$	$(0.27,0.9)$	$(0.78,0.6)$	$(0.78,0.6)$	$(0.43,0.92)$		$(0.78,0.6)$	$(0.99,0.2)$
$(0.52,0.78)$	$(0.41,0.84)$	$(0.94,0.37)$	$(0.71,0.66)$	$(0.94,0.37)$	$(0.78,0.6)$		$(0.94,0.37)$
$(0.92,0.43)$	$(0.65,0.7)$	$(0.52,0.78)$	$(0.27,0.9)$	$(0.11,0.96)$	$(0.99,0.2)$	$(0.94,0.37)$	

Table 3: Risk Table

Table 3 is the risk table indicating the risk involved in moving from the i^{th} to j^{th} node. The higher the membership value the more risky it is to transport goods between the nodes.

Table 4 is calculated with the help of table 2 and 3. The expense from moving from i^{th} to j^{th} node is a combination of the fuzzy cost and risk involved in moving from the i^{th} to j^{th} node. Expense is a weight function of both cost and risk with weight 0.6 and 0.4 respectively. For example :
$\tilde{C_{12}} \simeq 0.6\, \tilde{C'_{12}} \boxplus 0.4\, \tilde{r'_{12}}$ where $\tilde{C'_{12}}$ is the entry in $(1,2)$ cell in table 2 and $\tilde{r'_{12}}$ is the $(1,2)$ cell entry in table 3.
$\therefore \tilde{C_{12}} \simeq 0.6\,(0.8,0.48) \boxplus 0.4\,(0.41,0.84)$

$\tilde{C_{12}} \;\hat{=}\; (0.71, 0.60)$
Similarly all the $\tilde{C_{ij}}$ are calculated in table 4.

	$(0.71, 0.60)$	$(0.93, 0.17)$	$(0.23, 0.92)$	$(0.66, 0.78)$	$(0.84, 0.39)$	$(0.39, 0.89)$	$(0.90, 0.32)$
$(0.71, 0.60)$		$(0.94, 0.17)$	$(0.84, 0.39)$	$(0.73, 0.71)$	$(0.2, 0.95)$	$(0.71, 0.6)$	$(0.49, 0.84)$
$(0.93, 0.17)$	$(0.94, 0.17)$		$(0.63, 0.7)$	$(0.82, 0.42)$	$(0.86, 0.37)$	$(0.79, 0.65)$	$(0.50, 0.83)$
$(0.23, 0.92)$	$(0.84, 0.39)$	$(0.63, 0.7)$		$(0.87, 0.35)$	$(0.93, 0.17)$	$(0.70, 0.65)$	$(0.81, 0.43)$
$(0.66, 0.78)$	$(0.73, 0.71)$	$(0.82, 0.42)$	$(0.87, 0.35)$		$(0.71, 0.62)$	$(0.83, 0.57)$	$(0.42, 0.90)$
$(0.84, 0.39)$	$(0.2, 0.95)$	$(0.86, 0.37)$	$(0.93, 0.17)$	$(0.71, 0.62)$		$(0.86, 0.37)$	$(0.91, 0.51)$
$(0.39, 0.89)$	$(0.71, 0.6)$	$(0.79, 0.65)$	$(0.70, 0.65)$	$(0.83, 0.57)$	$(0.86, 0.37)$		$(0.79, 0.65)$
$(0.90, 0.32)$	$(0.49, 0.84)$	$(0.50, 0.83)$	$(0.81, 0.43)$	$(0.42, 0.90)$	$(0.91, 0.51)$	$(0.79, 0.65)$	

Table 4: Expense Table

5.2 SOLUTION

5.2.1 Step 1

We calculate the favourability of each customers as described in section 4.1
We take the weight as
$\alpha_1 = 0.4,\ \alpha_2 = 0.4,\ \alpha_3 = 0.2$
Now we calculate

$$F(\alpha_1 \tilde{P_i},\ \ \alpha_2 \tilde{W_i},\ \ \alpha_3 \tilde{C_{oi}}) = a_i^3 - a_i^{'3} + b_i^{'3} - b_i^3$$

Where

$$\alpha_1 \tilde{P_i} \;\boxplus\; \alpha_2 \tilde{W_i} \;=\; (a_i, b_i)$$
$$\alpha_3\, \tilde{C_{oi}} \;=\; (a_i^{'}, b_i^{'}) \quad \forall i \;=\; 1, 2, \ldots, n$$

For customer 1,

$\tilde{P_i} \;=\; (0.86, 0.36) \quad \tilde{W_i} \;=\; (0.66, 0.71) \quad \tilde{C_{oi}} \;=\; (0.8, 0.48)$
$\therefore\;$ 0.4 (0.86,0.36) $\boxplus$ 0.4 (0.66,0.71) $\hat{=}\; (0.69, 0.66) \;\boxplus\; (0.50, 0.87) \;\hat{=}\; (0.74, 0.57)$
$\alpha_3\, \tilde{C_{oi}} \;\hat{=}\; 0.2\ (0.8, 0.48) \;\hat{=}\; (0.511, 0.863)$
$\therefore\;\; F_1 \;\hat{=}\; (0.732740)$

Similarly for customer 2:

$\alpha_1 \tilde{P_2} \;\boxplus\; \alpha_2 \tilde{W_2} \;\hat{=}\; 0.4(0.52, 0.85) \;\boxplus\; 0.4(0.3, 0.97)$
$\alpha_1 \tilde{P_2} \;\boxplus\; \alpha_2 \tilde{W_2} \;\hat{=}\; (0.38, 0.93) \;\boxplus\; (0.22, 0.98)$
$\alpha_1 \tilde{P_2} \;\boxplus\; \alpha_2 \tilde{W_2} \;\hat{=}\; (0.41, 0.92)$

$\alpha_3\, \tilde{C_{o3}} \;\hat{=}\; 0.2(0.97, 0.08)$

$$\alpha_3\, \tilde{C_{o3}} \;\widetilde{=}\; (0.72, 0.60)$$
$$F_2 \;=\; -0.8904$$

Similarly calculating we get the favorabilities as follows:
$F_1 = 0.7327,\ F_2 = -0.8904,\ F_3 = 1.6495,\ F_4 = 1.0296,\ F_5 = 0.8673,\ F_6 = 0.8932,\ F_7 = 0.9720$

$l\ = (\ CUS_3,\ CUS_4,\ CUS_7,\ CUS_6,\ CUS_5,\ CUS_1)$
is the final list. And

$$Overall\ profit\ satisfaction \;=\; \sum_{i=1}^{7} \tilde{P_i};\ \ i \neq 2$$
$$\therefore\ \ Overall\ Profit\ Satisfaction \;=\; (0.99, 0.0049)$$

This concludes the step 1 of the solution process. Now foe step 2 we propose the following algorithms in sections 5.2.2, 5.2.3, 5.2.4

5.2.2 Fermatean Fuzzy Clarke Wright Algorithm

We calculate the savings list using the function given in 4.3.1 and calculating the list S_{ij} we have .
We find S_{13} as follows :

$$\tilde{C_{o1}} \;\boxplus\; \tilde{C_{o2}} \;\widetilde{=}\; (0.71, 0.60) \;\boxplus\; (0.23, 0.92)$$
$$\tilde{C_{o1}} \;\boxplus\; \tilde{C_{o2}} \;\widetilde{=}\; (0.71, 0.60) \;\boxplus\; (0.71, 0.552)$$

$$\tilde{C_{13}} \;\widetilde{=}\; (0.84, 0.39)$$

By equation (8)

$$S_{13} \;=\; (0.71)^3 - (0.84)^3 + (0.39)^3 - (0.552)^3$$
$$S_{13} \;=\; -0.1181 \;=\; S_{31}$$

Similarly Calculating

$S_{ij}\ \forall\ 1\ \leq\ i\ \leq 7\ \ and\ \ i+1\ \leq\ j\ \leq 7\ ;\ i,\ j\ \neq\ 2$

and sort the list in descending order we get the list as:

$\{\ S_{37},\ S_{45},\ S_{17},\ S_{47},\ S_{57},\ S_{36},\ S_{67},\ S_{56},\ S_{34},\ S_{46},\ S_{14},\ S_{15},\ S_{13},\ S_{35}\}$

Now, using the above list we create the following route, taking $k = 3$ customers at a time.

1. $0-3-7-1-0$
2. $0-4-5-6-0$

$\therefore$ trip 1 : $\tilde{C_{03}} \;\boxplus\; \tilde{C_{37}} \;\boxplus\; \tilde{C_{71}} \;\boxplus\; \tilde{C_{10}}$

Trip 1 expense, $\tilde{C_1} \;\widetilde{=}\; (0.90, 0.199)$

Similarly

Trip 2 expense, $\tilde{C_2} \ \hat{=} \ (0.94, 0.159)$

Total Expense , $\tilde{C} \ \hat{=} \ (0.90, 0.199) \ \boxplus \ (0.94, 0.159)$ $\tilde{C} \ \hat{=} \ (0.98, 0.03)$

Final Result

$\hat{=} \ \alpha\tilde{P} \ \boxminus \ \tilde{C}$; where $\alpha \ = \ 1$
$\hat{=} \ (0.99, 0.00049) \ \boxminus \ (0.98, 0.03)$
$\hat{=} \ (0.79, 0.01)$

Fuzzy Cost Of Transportation $\ \hat{=} \ \tilde{40} \ (0.994, 0.014)$

5.2.3 Fermatean Fuzzy Nearest Neighbourhood

We initiate route by searching for least expensive route from depo node O in table 4 :
min $\{\tilde{C}_{0j}; 1 \leq j \leq 7; j \neq 2\}$ = (0.23,0.92) = $\tilde{C}_{03}$
Here $j \neq 2$ as customer 2 is rejected.
We eliminate $\tilde{C}_{03}$ and $\tilde{C}_{30}$ from matrix $\tilde{C}_{0j}$
initiate route 1: O-3
expense ; $\tilde{C} = \tilde{C}_{0,3}$
from node 3 we find
min$\{\tilde{C}_{3j}; 1 \leq j \leq 7; j \neq 2, 0\}$ = (0.70, 0.65) = $\tilde{C}_{36}$
$\therefore$ the route is extended ; route 1: $0-3-6$
$\tilde{C} \ = \ \tilde{C}_{03} \boxplus \tilde{C}_{36}$
similarly proceeding in this way we get the next cheap node as $\tilde{C}_{61}$
$\therefore$ the route is $0-3-6-1-0$ = TRIP 1;
$\tilde{C}_1 = \tilde{C}_{03} \boxplus \tilde{C}_{36} \boxplus \tilde{C}_{61} \boxplus \tilde{C}_{10}$ = (0.90, 0.215)
similarly eliminating entries from matrix $\tilde{C}_{ij}$:
$\tilde{C}_{ij}$: $i = 1,3,6$ *and* $j = 1,3,6$
and repeating the process with the updated $\tilde{C}_{i,j}$ we get the next route as
Route 2: 0-4-7-5-0
$\tilde{C}_2 \ = \ \tilde{C}_{04} \boxplus \tilde{C}_{47} \boxplus \tilde{C}_{75} \boxplus \tilde{C}_{50} \ = \ (0.97, \ 0.139)$
Total expense, $\tilde{C} \ = \ (0.993, \ 0.139)$
Final Result $\hat{=} \ (0.99, 0.00049) \ \boxminus \ (0.993, 0.29)$ as $\alpha \ = \ 1$
= (0.52, 0.016)
Fuzzy Cost Of Transportation $\ \hat{=} \ \tilde{43} \ (0.998, 0.0043)$

5.2.4 FF Nearest Neighbourhood + Brute Force

The selected customer list from the section 5.2.1 is
l= $\{$cus3 ,cus4 ,cus7 ,cus6 ,cus5 ,cus1$\}$
then the group by FFNN methods in section 5.2.3 are (1,3,6) (4,5,7)
Now for the group (1,3,6) we have $3! = 6$ possible different route combinations as follows;

1. $0-1-3-6-0$
2. $0-1-6-3-0$

3. $0-3-1-6-0$

4. $0-3-6-1-0$

5. $0-6-3-1-0$

6. $0-6-1-3-0$

we calculate the expense for each route and select the route with minimum expense.
For example for expense of (1) : 0-1-3-6-0 is
$\tilde{C}_{01} \boxplus \tilde{C}_{13} \boxplus \tilde{C}_{36} \boxplus \tilde{C}_{60} = (.942, 135)$
similarly for others also we get
(2) 0-1-6-3-0 = (0.90,0.215)
(3) 0-3-1-6-0 = (0.911,0.191)

NOTE:

$$0-1-3-6-0 \; and \; 0-6-3-1-0$$
$$0-1-6-3-0 \; and \; 0-3-6-1-0$$
$$0-3-1-6-0 \; and \; 0-6-1-3-0$$

are routes exactly opposite to each other hence have same expense level.
least expensive route is : 0-1-6-3-0 or 0-3-6-1-0 with expense : $\tilde{C}_1 = (0.90, 0.215)$

similarly for the other group (4,5,7) the possible combinations are

1. $0-4-5-7-0$

2. $0-4-7-5-0$

3. $0-5-4-7-0$

4. $0-5-7-4-0$

5. $0-7-4-5-0$

6. $0-7-5-4-0$

And the least expensive route is

$$0-7-4-5-0 \; or \; 0-5-4-7-0$$

with expense $\simeq (0.97, 0.139)$

Total Expense

$\tilde{C} \simeq (0.90, 0.215) \boxplus (0.97, 0.139)$
$\tilde{C} \simeq (0.993, 0.29)$

Result $\simeq (0.52, 0.016)$
Fuzzy Cost Of Transportation $\simeq \tilde{43} \; (0.998, 0.0043)$

6 Comparision Of Results

6.1 Among Sections 5.5.2, 5.2.3, 5.2.4

	Selected Customers	Trips		Result	Fuzzy Cost	Expected Profit
FFCWA	$l = \{3,4,7,6,5,1\}$	$0-3-7-1-0$	$0-4-5-6-0$	$(0.79, 0.001)$	$\tilde{40}(0.994, 0.014)$	$\tilde{201}(0.9407, 0.35)$
FFNN	$l = \{3,4,7,6,5,1\}$	$0-3-6-1-0$	$0-4-7-5-0$	$(0.52, 0.016)$	$\tilde{43}(0.998, 0.0043)$	$\tilde{198}(0.793, 0.506)$
FF(NN+B)	$l = \{3,4,7,6,5,1\}$	$0-3-6-1-0$	$0-4-7-5-0$	$(0.52, 0.016)$	$\tilde{43}(0.998, 0.0043)$	$\tilde{198}(0.793, 0.506)$

Table 5:

The step 1 of the solution procedure was same for all the algorithms. Out of 7 possible customers we chose top 6 most customers attractive or favorable customers in section 5.2.1 using the procedure explained in section 4.1 We saw for the above example in section 5, the FFCLA gave us a better result (i.e section 5.2.2) the then FFNN and FF(NN+B). The route given by FFCLA is:
TRIP 1 : 0-3-7-1-0
TRIP 2 : 0-4-5-6-0
with the overall satisfaction level (0.79, 0.01) which is higher then the satisfaction level given by FFNN and FF(NN+B). For the above example we saw that both the methods FFNN and FF(NN+B) gave us the same route
TRIP 1: 0-1-6-3-0
TRIP 2: 0-7-4-5-0
with the overall satisfactory level as (0.52, 0.16).
However in general FFNN and FF(NN+B) may not provide the same set of solutions. In general FF(NN+B) give us a route route which is either same as FFNN or less expensive the FFNN.
Also the fuzzy expected cost is calculated from the table 1
we saw that the transportation Fuzzy cost under FFCWA is $\tilde{40}$ (0.994,0.014). For the FFNN and FF(NN+B) the fuzzy cost is
$\tilde{43}$ $(0.998, 0.0043) \geq \tilde{40}$ (0.994,0.014)

6.2 FFMVRPP AND CLASSICAL MVRPP

Let us now have a comparison between the VRPP in FF environment and in deterministic environment. In deterministic environment the VRPP models takes only two factors cost and profit while in FF environment the FFMVRPP models includes cost, profit, risk and willingness variables, clearly having a more realistic and practical approach for decision making. The willingness or relation of customer with seller/distributor plays a very dominant role in practical scenerios which is completely ignored in the deterministic approach of the VRPP.

Under Fermatean Fuzzy Environment	Under Deterministic Environment
FFCWA **Trips** 0-3-7-1-0 0-4-5-6-0 Cost: $\widetilde{40}$(0.994, 0.014) Profit: $\widetilde{201}$(0.9407, 0.35) Overall Risk: (0.89, 0.0527)	**CWA** **Trips** 0-1-5-7-0 0-6-4-3-0 Cost: 40 Profit: 20 Overall Risk: (0.99, 0.00928)
FFNN and FF(NN+B) **Trips** 0-1-6-3-0 0-7-4-5-0 Cost: $\widetilde{43}$(0.998,0.0043) Profit: $\widetilde{198}$(0.793, 0.506) Overall Risk: (0.8724, 0.1142)	**NN and (NN+B)** **Trips** 0-4-1-6-0 0-3-7-5-0 Cost: 38 Profit: 203 Overall Risk: (0.999, 0.200)

Table 6: FFMVRPP vs MVRPP

From the above table it can be clearly seen that even for the same set of customers we get different Groups and routes for transporting. In FF environment by FFCWA we get the trips as

$$0-1-5-7-0 \textit{ and } 0-6-4-3-0$$

with the profit $\tilde{2}01$ $(0.994, 0.014)$ and overall transportation risk (0.89,0.052). For the deterministic case by CWA we get the routes as

$$0-3-7-1-0 \textit{ and } 0-4-5-6-0$$

with similar profits 201. But if we calculate the risk of this trip then it comes out to be (0.99,0.00928) which is higher then that of FFCWA.

Similarly for NN and NN+B also, we get the routes in deterministic environment as

$$0-4-6-1-0 \textit{ and } 0-3-7-5-0$$

with a higher profit of 203 units. In the FF environment the method gives us the routes as

$$0-3-6-1-0 \textit{ and } 0-4-7-5-0$$

with a lesser profit of $1\tilde{9}8$ $(0.793, 0.506)$. But when compared with the risk factor we can see that in FF environment the risk is less $(0.872, 0.1142)$ then that of the deterministic environment $(0.999, 0.200)$. Now in practical situations risk plays a more dominant role then profit as most of the firms prefers less risk even if the cost is a bit higher.

NOTE: If we remove the willingness and risk factors from our FFMVRPP then we would get similar result i.e we would get similar kind of set of customers and routes. The deviation is due to the extra qualitative factors that we have included in our model showing the effect of these factors.

7 Conclusion and Future Objectives

In this paper FFMVRPP has been proposed. For the solution a two step procedure has been proposed in which the first step deals with the selection and in second routes and trips are created for visiting customers. The end result comes out to be a fermatean fuzzy value indicating the satisfaction level achieved through the whole process. Expected fuzzy cost and profits can also be calculated if desired.

It can be clearly seen that FFMVRPP is much more realistic and practical when compared to the deterministic MVRPP. Considering the factor “Relation of a customer with seller” in step 1 or in selecting customers is very important as in real life situations this factor plays a very important and dominant role in the decision making which is completely ignored in the deterministic model. Also including the risk factor for selection of routes is very important as this qualitative factor also is very important in the real situations. So it can be concluded that decision making under the FF environment is much more efficient when compared to the Rigid or deterministic environment.

In future research,we are working to modify the methodologies to get an optimal solution of the FFMVRPP. Also it can be extended to solve FFMVRPP having different data structure i.e when the data is given in interval valued or any other type of Fermetean Fuzzy numbers.

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