Impulse Elimination for Singular Second-Order System: Approach

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***Abstract*—This paper considers approaches to partial eigenvalue assignment in second-order descriptor systems via proportional plus derivative plus output feedback controller. The impulse elimination approach by output feedback control is addressed by combining the eigenstructure and the closed-loop system's finite eigenstructure. More precisely, based on the desired eigenstructure, the gains controller's parametric expressions making the closed-loop system impulse-free and assigning the finite eigenstructure are formulated. The simulation results are provided to verify the effectiveness of the proposed method. This study presents an approach to partial eigenvalue assignment for the descriptor system where an algorithm is presented for calculated the output feedback matrix by the Sylvester equation. Sylvester equations present the theorems. Two algorithms are implemented using the Sylvester equation, and examples were presented with finally their conclusions.**

Keywords: Impulse elimination, Second-order system, Sylvester equation.

I. Introduction

Second-order linear systems have found wide applications in many scientific and engineering fields, such as the control of large flexible space structures, earthquake engineering, robotics control, control of mechanical multibody systems, and vibration control in structural dynamics. The model of second-order system has aroused great interest and a wide variety of practical applications over the last few decades [1], [2]. There are many mathematical methods for solving model and designing controllers [3], [4], [5], [28], [9], [10], [11]. It is well known that the eigenstructure assignment (ESA) and pole assignment are effective approaches to improve the dynamic characteristics of singular second-order system, including stabilisation, impulse elimination and decoupling [5], [28], [9], [10], [11] [12], [13], [14] . The ESA based on state and derivative feedback control for second-order systems has many valuable results [5], [28], [9], [10], [11] [18], [15] [19] [20].

A generalized Sylvester equation's solution is associated with a linear descriptor system and subject to some rank and regional pole-placement constraints. Under the hypothesis of strong-detectability of the descriptor system, a sequence of coordinate transformations is proposed such that the considered problem can be solved through a Sylvester equation is associated with a detectable reduced-order normal system [37].

The output regulation for second order system via feedback is presented in [16] [17]. The eigenvalue assignment with minimum sensitivity for second-order systems via proportional–derivative state feedback is proposed in [21]. The parametric approach for ESA in second-order system via velocity plus-acceleration feedback is presented in [28], [9], [10], and then, the parametric expressions of gain controllers assigning the eigenstructure via velocity-plus-acceleration feedback are formulated. The partial pole assignment [22], [23] and partial eigenstructure assignment [24] [25] are concerned by various approaches. Impulsive behavior is an important characteristic of descriptor systems. Impulse terms may destroy the system and hence are expected to be eliminated in descriptor systems. There are many approaches and results of impulse elimination for descriptor systems [11] [26] [27] [28] . The impulsive modes can be eliminated in descriptor systems via state feedback and output feedback [11] [28]. The impulse controllability and impulse observability are necessary conditions for impulse elimination in descriptor systems [28] [29], [31], [32]. The impulsive mode controllability is proposed for descriptor systems in [30], and the criteria of impulsive mode controllability are established. A structured output proportional and derivative feedback approach is presented for the problem of impulsive modes elimination in descriptor systems. Disturbance impulse controllability for the descriptor system is introduced in [32]. The controllability and observability conditions of second-order linear systems are analyzed in [33]. The impulse elimination problems for second-order systems

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have not been investigated in the literature. In this paper, the impulse elimination problem is investigated, respectively, via a class of feedback controllers by the ESA approach for a singular second-order system. The solvability of the proposed problem is given by analyzing the desired eigenstructure. Under solvability conditions, the complete parametric expressions for controller gains of normalisation and impulse elimination are derived, respectively. Finally, illustrating examples are given.

Consider the control of the following second-order descriptor dynamical linear system:

$$\begin{aligned} M\ddot{x} + D\dot{x} + Nx &= Bu \\ y_0 &= C_0 x \\ y_1 &= C_1 \dot{x} \\ y_2 &= C_2 \dot{x} \end{aligned} \tag{1}$$

where $x \in R^n$ and $u \in R^m$ are the state vector and the control vector, respectively, and $M, D, N \in R^{n\times n}$, and $B \in R^{n\times m}$, $C_0, C_1, C_2 \in R^{p\times n}$ are the system coefficient matrices. In certain applications, the matrices M, D, and N being called the mass matrix, the structural damping matrix and the stiffness matrix, respectively [10] [6], [7], [8] . We know that singular system may contain impulse terms in its state solution. Impulse elimination is to design a controller such that the state solution of resulted closed-loop system has no impulse terms. In [34] the condition for impulsive-mode controllability is derived and proved to be necessary and sufficient for the existence of admissible feedback controls eliminating impulsive modes.

The article present in section II the problem formulation and preliminaries, in section III present the impulse elimination, in section IV present the problem eigenstructure assignment , and the section V the conclusions.

II. Problem formulation and preliminaries

These coefficient matrices satisfy the following assumptions.

Assumption 2.1: A_1: rank(M) = q, $0 < q \leq n$, rank(B) = m, and $rank(C_0) = rank(C_1) = rank(C_2) = p$.

For the second-order descriptor dynamical system (1), by choosing the following control law:

$$u(t) = -F_0 y_0(t) - F_1 y_1(t) - F_2 y_2(t) \tag{2}$$

with $F_0, F_1, F_2 \in R^{p\times n}$. For feedback controller in the equation (2), these conditions will no longer be needed. Besides, the impulse elimination problem that we will study for singular second-order system is not considered in [5] [28] [9] [10] [6], [7] [8]. By feedback controller in the equation (2), the system (1) can be transformed into the closed-loop system as follows:

$$(M + BF_2C_2)\ddot{x} + (D + BF_1C_1)\dot{x} + (N + BF_0C_0)x = 0 \tag{3}$$

The corresponding quadratic polynomial matrix is

$$P(\lambda) = \lambda^2(M + BF_2C_2) + \lambda(D + BF_1C_2) + N + BF_0C_0 \tag{4}$$

The system (3) can be written in the first-order state-space form

$$E_c\dot{z} = A_c z; \tag{5}$$

with

$$\begin{aligned} E_c &= \begin{bmatrix} I_n & 0 \\ 0 & (M + BF_2C_2) \end{bmatrix} \quad and \\ A_c &= \begin{bmatrix} 0 & I \\ -(N + BF_0C_0) & -(D + BF_1C_1) \end{bmatrix} \end{aligned} \tag{6}$$

For feedback controller in the equation (2), these conditions will no longer be needed. Besides, the impulse elimination problem that we will study for singular second-order system is not considered in [5] [28], [9], [10], [6], [7], [8].

Simultaneously, rewrite the equation (2) as

$$u = -\begin{bmatrix} F_0 & F_{11} \end{bmatrix}\dot{z} - \begin{bmatrix} F_{12} & F_2 \end{bmatrix} z$$

where $F_{11} + F_{12} = F_1$. Substituting this into (2), we have

$$E_c\dot{z} = A_c z \tag{7}$$

with

$$\begin{aligned} E_e &= \begin{bmatrix} I_n & 0 \\ BF_{12}C_1 & M + BF_2C_2 \end{bmatrix}; \\ A_c &= \begin{bmatrix} 0 & I \\ -(N + BF_0C_0) & -(D + BF_{11}C_1) \end{bmatrix} \end{aligned} \tag{8}$$

$\dot{z} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$ and $z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

Definition 2.1: The second-order dynamical system (1), is called S-controllable if and only if the corresponding extended first-order state-space representation (5), (6), is S-controllable. The objective is to design the feedback controller (2) such that the closed-loop system has $2n$ eigenvalues. This implies

$$degdet[\lambda^2(M + BF_2C_2) + \lambda(D + BF_1C_1) + N + BF_0C_0] = 2n \tag{9}$$

III. Impulse elimination

In [34] prove the equivalence between impulsive-mode controllability and eliminating impulsive modes by feedback control. After impulsive modes being removed by feedback control, a nonsingular descriptor system has a unique smooth solution for each initial admissible condition and hence is free of impulse. Therefore, impulsive-mode elimination and impulse elimination are two different problems. The latter is equivalent to enabling the controlled system to have no impulsive modes and be nonsingular. In [30], impulsive behavior is an important issue in the descriptor system, and impulse terms are not expected to exist. A system is called impulse-free, or equivalently, having no impulsive modes if there is no impulse term in the state solution. Similarly, the response of the singular second-order system (1) may contain impulse terms. Consider the problem of impulse elimination for system (1) via feedback controller (2). By using the ESA method, we present the parametric expressions of gain controllers, ensuring the closed-loop system impulse-free. The following lemma, which is obtained directly by the results of [35], gives a simple rank criterion for a second-order system to be impulse-free.

Lemma 3.1: [30] The system (1) is impulse-free if and only if

$$rank\begin{bmatrix} M & D & N \\ 0 & M & D \\ 0 & 0 & M \end{bmatrix} = rank\begin{bmatrix} M & D \\ 0 & M \end{bmatrix} = n \tag{10}$$

This result provides an approach to detect impulsive modes of system (1). In this section, the impulse elimination problem is investigated based on the following lemma instead of Lemma 3.1.

Lemma 3.2: If the system (5), (6) is impulse-free, then the system (3) is impulse-free.

Lemma 3.2 is easy to prove by matrix decomposition. By Lemma 3.2, in this section, we consider impulse elimination in the system (5), (6) by feedback controller (2) . The objective is to design the controller (2) such that system (5), (6) is impulse-free. For the system (7), (8) we know that $rankE \geq degdet(sE-A)$ and closed-loop system is impulse-free if and only if $rankE = degdet(sE-A)$,

$$E = \begin{bmatrix} I & 0 \\ BF_{12}C_1 & M+BF_2C_2 \end{bmatrix} \tag{11}$$

$$A = \begin{bmatrix} 0 & I \\ -(N+BF_0C_0) & -(D+BF_{11}C_1) \end{bmatrix} \tag{12}$$

We assume that $n_1 = n + rank[M,B]$ is the number of desired finite eigenvalues, and $n_2 = 2n - n_1$. Then

$$\begin{aligned} n_1 = degdet(sE-A) \leq rankE = n \\ +rank(M+BF_2C_2) \leq n + rank[M,B] \end{aligned} \tag{13}$$

Therefore, $rankE = degdet(sE-A)$, and the closed-loop system is impulse-free. Based on ESA approach and Lemma 3.2, parametric expressions of gains controller making the closed-loop system impulse-free and assigning the finite eigenstructure (i.e. Jordan structure of finite eigenvalues) of the closed-loop system are derived.

$$\begin{gathered} J_1 = diag(J_1, J_2, J_3, \cdots, J_p) \\ J_i = diag(J_{i1}, J_{i2}, \cdots, J_{iq_i}) \\ J_{ij} = \begin{bmatrix} \lambda_i & 1 & \cdots & \\ 0 & \lambda_i & 1 & \cdots \\ 0 & \cdots & \lambda_i & 1 \\ 0 & \cdots & \cdots & \lambda_i \end{bmatrix} \end{gathered} \tag{14}$$

where $J_{ij}, j = 1, 2, \cdots, q_i$, are the q_i Jordan blocks associated with the eigenvalue λ_i. J is the Jordan matrix associated with all the finite eigenvalues of closed-loop system.

The results are presented by the following theorem.

Theorem 3.1: Assume that system (1) is $S-$controllable. Then based on the prescribed finite eigenstructure with the form of (14), the controller gains for impulse elimination and arbitrary finite ESA by feedback controller (2) are as

$$\begin{aligned} \begin{bmatrix} F_{12} & F_2 \end{bmatrix} = \begin{bmatrix} H_fJ^{-1} & H_\infty \end{bmatrix} V^{-1} \\ \begin{bmatrix} F_0 & F_{11} \end{bmatrix} = \begin{bmatrix} R_f & R_\infty \end{bmatrix} V^{-1} \end{aligned} \tag{15}$$

where $V \equiv \begin{bmatrix} V_f & V_\infty \end{bmatrix}$, $V_f \in C^{2n\times n_1}$, $H_f, R_f \in C^{r\times n_1}$

$$V_\infty = \begin{bmatrix} 0 \\ \tilde{V}_\infty \end{bmatrix} \in C^{2n\times n_2} \tag{16}$$

$H_\infty \in C^{r\times n_2}$ and

$$\begin{bmatrix} \tilde{V}_\infty \\ H_\infty \end{bmatrix} \in ker[M,B] \tag{17}$$

$R_\infty \in C^{r\times n_2}$ is any given matrix.

Proof:

Assume that system (1) is $S-$controllable. Based on the finite ESA results in Theorem 3 in [30], where is considered the equation (2) .

Write this as:

$$\begin{aligned} \begin{bmatrix} F_{12} & F_2 \end{bmatrix} V_fJ = H_f \\ \begin{bmatrix} F_0 & F_{11} \end{bmatrix} V_f = R_f \end{aligned} \tag{18}$$

Let

$$\begin{bmatrix} F_{12} & F_2 \end{bmatrix} V_\infty = H_\infty \tag{19}$$

then $H_\infty \in C^{r\times n_2}$ and

For given R_∞, let

$$\begin{bmatrix} F_0 & F_{11} \end{bmatrix} V_\infty = R_\infty \tag{20}$$

Combining (18), with the equations (19), and (20), we obtain

$$\begin{aligned} \begin{bmatrix} F_{12} & F_2 \end{bmatrix} = \begin{bmatrix} H_fJ^{-1} & H_\infty \end{bmatrix} V^{-1} \\ \begin{bmatrix} F_0 & F_{11} \end{bmatrix} = \begin{bmatrix} R_f & R_\infty \end{bmatrix} V^{-1} \end{aligned} \tag{21}$$

where

$$V \equiv \begin{bmatrix} V_f & V_\infty \end{bmatrix} = \begin{bmatrix} \bar{V}_f & 0 \\ \tilde{V}_f & \tilde{V}_\infty \end{bmatrix}. \tag{22}$$

The proof is therefore completed.

IV. Problem Eigenstructure Assignment

Consider the following linear time-invariant descriptor system

$$E\dot{x}(t) \quad = \quad Ax(t) + Bu(t); y(t) = Cx(t) \tag{23}$$

where $E, A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$ and rank$E = q \leq n$. Assume that the matrix pencil$(\lambda E - A)$ is nonsingular, i.e., rank$(\lambda E - A) = n$.

Proposition 4.1: [39] The system (23) is controllable at infinity if and only if

$$rank[E,B] = rank[E,A,B] \tag{24}$$

Proposition 4.2: [39] The system (23) is C-controllable if and only if condition (24) is satisfied together with

$$rank[\lambda E - A, B] = rank[E, A, B], \forall \lambda \in C. \tag{25}$$

Proposition 4.3: [39] The system (23) is I-controllable if and only if

$$\begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} = rank[E, A, B] + rankE. \tag{26}$$

Proposition 4.4: [39] The system (23) is S-controllable if and only if both the conditions (25) and (26) are satisfied.

A. Eigenstructure by equation Sylvester

The system (3) can be written in the first-order state-space form (7) and (8). Thus for obtained the output feedback K $\sigma(E_d, A_d + B_d K C_d) \in C^-$, is used the Silvester equation in [38], [36].

Consider the following linear time-invariant descriptor system in [38], [36].

$$\begin{aligned} E_d \dot{x}(t) &= A_d x(t) + B_d u(t) \\ y(t) &= C_d x(t) \end{aligned} \tag{27}$$

The Sylvester equations in [38], [36].

$$\begin{aligned} A_d V_d - E_d V_d H_V &= -B_d W_d, \qquad \sigma(H_V) \in \mathcal{C}^- & (28) \\ P_d A_d - H_P P_d E_d &= -U_d C_d, \qquad \sigma(H_P) \in \mathcal{C}^- & (29) \end{aligned}$$

the theorem (4.1) is based , [38].

Theorem 4.1: Let (1), be S-controllable, and $V_d \in R^{2n\times p}$ and $W_d \in R^{m\times p}$ satisfy the equation (28). Then, the following hold.

1) The matrices V_d and W_d given by (28),

$$\begin{bmatrix} A_d - \lambda_i E_d & B_d \\ P_d E_d & 0 \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} = i = 1, 2, \cdots, q. \tag{30}$$

satisfy Sylvester matrix equation (28) for, $i = 1, 2, \cdots, q$,

2) When

$$rank(\begin{bmatrix} V_i \\ W_i \end{bmatrix}) = m \; i = 1, 2, \cdots, q. \tag{31}$$

hold, (30) gives all the solutions.

Proof Based in [38], [36].

The following basic procedure is proposed to calculate the feedback controller that stabilizes the closed loop system, when $m+p > q$. Closed loop eigenvalues are positioned arbitrarily close to the set ; they are symmetric sets of pre-specified eigenvalues. The (E_d, A_d, B_d, C_d) system is considered to be strongly controllable and strongly detectable.

Algorithm S_1

Step 1: Choose an array $H_P \in \Re^{q-p\times q-p}$ such that $\sigma(H_P) = \Lambda_P \in \mathcal{C}^-$ and Sylvester's equation (29) is solved to find a matrix

$P_d \in \Re^{n+q-p\times n+q}$ such that

$$rank\left(\begin{bmatrix} P_d E_d \\ C_d \end{bmatrix}\right) = q \tag{32}$$

Step 2: Sylvester's equation (28) is solved, for some $H_V \in \Re^{p\times p}$ matrix such that $\sigma(H_V) = \Lambda_V \in \mathcal{C}^-$ taking into account that the matrix V_d must taking into account that $rank\,(E_d V_d) = p$ (or $Ker\,(P_d E_d) = Ker\,(E_d) \oplus Im\,(V_d)$, where $\oplus$ represents the direct sum).

Step 3: By construction, the matrix V_d must verify that rank $(C_d V_d) = p$ and the matrix K can be calculated by:

$$K = W_d (C_d V_d)^{-1} \tag{33}$$

∘

Remark 4.1: steps 1 and 2 can be solved using standard techniques for positioning the self-structure. Similar to the previous case, the matrices V_d and W_d used for the calculation of K can be constructed only with real elements. In particular: if $\lambda_i \in \mathcal{C}$, it is considered $\lambda_{i+1} = \lambda_i^*$ e $\begin{cases} V_i = Re\,(v_i), & V_{i+1} = Imag\,(v_i) \\ W_i = Re\,(w_i), & W_{i+1} = Imag\,(w_i) \end{cases}$, where V_i and W_i denote the columns of the matrices V_d and W_d, respectively. In step 1, under the condition that the system is strongly observable (detectable). As will be seen later, degrees of freedom existing in the choice of V_d that satisfy the coupling condition $P_d E_d V_d = 0$, can also be used to guarantee obtaining K such that $K C_d V_d = W_d$ in [38].

B. Example

Consider a simple linear dynamical system (1) in [30]

$$M = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 25 & -15 & 0 & 0 \\ -15 & 35 & -20 & 0 \\ 0 & -20 & 60 & -40 \\ 0 & 0 & -40 & 40 \end{bmatrix}$$

$$N = \begin{bmatrix} 15 & -10 & 0 & 0 \\ -10 & 25 & -15 & 0 \\ 0 & -15 & 35 & -20 \\ 0 & 0 & -20 & 20 \end{bmatrix} \; B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

Considered the system in the equations (7), (8)

$$E_d = \begin{bmatrix} I_n & 0 \\ 0 & M \end{bmatrix}; A_d = \begin{bmatrix} 0 & I_n \\ -N & -D \end{bmatrix};$$

$$B_d = \begin{bmatrix} 0 \\ B \end{bmatrix} \; C_d = \begin{bmatrix} B' & 0 \end{bmatrix}$$

Algorithm $S1$

Resolved the equation (28) for calculate the matrices W_d, V_d, satisfies the equation (31) and the matrix K, such that $K C_d V_d = W_d$:

$$V_d = \begin{bmatrix} 0.0266594 & 0.012664 \\ 0.0532409 & 0.0292111 \\ 0.0526191 & 0.0275588 \\ 0.0428678 & 0.0204925 \\ -0.0799783 & -0.0506558 \\ -0.1597228 & -0.1168444 \\ -0.1578572 & -0.1102353 \\ -0.1286034 & -0.0819702 \end{bmatrix}$$
$$W_d = \begin{bmatrix} 0.9836714 & 0.9919888 \\ 0.9751273 & 0.9892803 \end{bmatrix}$$
$$K = \begin{bmatrix} -172.58241 & 135.085 \\ -173.19315 & 135.43792 \end{bmatrix}$$ where the eigenvalues are $\lambda_1 = -3$, $\lambda_2 = -4$, $\lambda_3 = -5.261107$, $\lambda_4 = -1.5034249$, $\lambda_5 = -0.509633$, $\lambda_6 = -0.7730653$, $\lambda_7 = -22.229984$, $\lambda_8 = -0.229984$.

C. Numerical algorithm

We first present an approach to the general solutions for $F_0, F_1 and F_2$. Let Q denote the matrix that its rows are comprised of orthonormal basis vectors of the null space, we have

$$[F_0, F_1, F_2] = VQ \tag{34}$$

where the parametric matrix V is to be determined based in the sylvester equation (28).

F_0, F_1 and F_2 must also implement some given eigenvalues assignment.

The theorem 4.2 is based in , [40] [38].

Theorem 4.2: Let (1), be S-controllable, and $V_d \in R^{2n\times p}$ and $W_d \in R^{m\times p}$ satisfy the equation (28). Then, the following hold.

1) The matrices V_d and W_d given by (35),

$$\begin{bmatrix} A_d - \lambda_i E_d & B_d \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix} = \; i = 1, 2, \cdots, q. \tag{35}$$

satisfy Sylvester matrix equation (28) for, $i = 1, 2, \cdots, q$,

2) When

$$rank(\begin{bmatrix} V_i \\ W_i \end{bmatrix}) = m \; i = 1, 2, \cdots, q. \tag{36}$$

3) F_0, F_1, F_2 is such that it satisfies

$$(M + BF_2C_2)\ddot{q}(t) + (D + BF_1C_1)\dot{q}(t) + (N + BF_0C_0)q(t) = 0 \tag{37}$$

Proof Based in [37], [36], [38] and [40].

The following basic procedure is proposed to calculate the feedback controller that stabilizes the closed loop system, when $m+p > q$. Closed loop eigenvalues are positioned arbitrarily close to the set ; they are symmetric sets of pre-specified eigenvalues. The (E_d, A_d, B_d, C_d) system is considered to be strongly controllable and strongly detectable.

Algorithm $Z1$ Input: M, D, N, B, C_0, C_1, C_2 Output: F_0, F_1, F_2

Step (1) Compute the left null space Q of the coefficient matrix in Equation (23).

Step (2) Compute v_i, w_i, $i =, 01, 2$ by Equations (28), (35), (36) to form V_0, V_1, V_2 and W_0, W_1, W_2.

Step(3) With the matrices W_0, W_1, W_2, V_0, V_1, V_2, satisfies the equation (35),(36) and the matrix F_0, F_1,F_2 such that $F_2C_2V_2 = W_2$, $F_1C_1V_1 = W_1$, and $F_0C_0V_0 = W_0$,

Step (4) Substitute V back into Equation (34) with (37) to give F_0, F_1, F_2.

○

D. Example

Consider a simple linear dynamical system (1) in [30]

$$M = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$D = \begin{bmatrix} 25 & -15 & 0 & 0 \\ -15 & 35 & -20 & 0 \\ 0 & -20 & 60 & -40 \\ 0 & 0 & -40 & 40 \end{bmatrix}$$
$$N = \begin{bmatrix} 15 & -10 & 0 & 0 \\ -10 & 25 & -15 & 0 \\ 0 & -15 & 35 & -20 \\ 0 & 0 & -20 & 20 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \; C_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$
$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; \; C_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

Considered the system in the equations (4), (5)

$$E_d = \begin{bmatrix} I_n & 0 \\ 0 & M \end{bmatrix}; \; A_d = \begin{bmatrix} 0 & I_n \\ -N & -D \end{bmatrix};$$
$$B_d = \begin{bmatrix} 0 \\ B \end{bmatrix} \; C_d = \begin{bmatrix} B' & 0 \end{bmatrix}$$

Algorithm Z1

Step (1) Compute the left null space Q of the coefficient matrix in Equation (23).

Step (2) Compute v_i, w_i, $i =, 01, 2$ by Equations (28), (35), (36) to form V_0, V_1, V_2 and W_0, W_1, W_2.

Step(3) With the matrices W_0, W_1, W_2, V_0, V_1, V_2, satisfies the equation (35),(36) and the matrix F_0, F_1,F_2 such that $F_2C_2V_2 = W_2$, $F_1C_1V_1 = W_1$, and $F_0C_0V_0 = W_0$,

Step (4) Substitute V back into Equation (34) with (37) to give F_0, F_1, F_2.

$$V_0 = V_1 = V_2 = \begin{bmatrix} 0.0266594 & 0.012664 \\ 0.0532409 & 0.0292111 \\ 0.0526191 & 0.0275588 \\ 0.0428678 & 0.0204925 \\ -0.0799783 & -0.0506558 \\ -0.1597228 & -0.1168444 \\ -0.1578572 & -0.1102353 \\ -0.1286034 & -0.0819702 \end{bmatrix}$$
$$W_0 = W_1 = W_2 = \begin{bmatrix} 0.98367140.9919888 \\ 0.97512730.9892803 \end{bmatrix}$$
$$F_0 = \begin{bmatrix} -6494.8334 & 10556.91 \\ -6511.9614 & 10584.49 \end{bmatrix}$$

where the eigenvalues are
$\lambda_1 = -3$, $\lambda_2 = -4$, $\lambda_3 = -26.609165$, $\lambda_4 = -0.4710995$, $\lambda_5 = -0.7617854$.

$$F_1 = \begin{bmatrix} -230.40991 & 133.84942 \\ -231.41824 & 134.19385 \end{bmatrix}$$

where the eigenvalues are $\lambda_1 = -3$, $\lambda_2 = -4$, $\lambda_3 = -22.523982$, $\lambda_4 = -0.5$, $\lambda_5 = -0.7672673$, $\lambda_6 = -2.7710422 + 0.7706989j$, $\lambda_7 = -2.7710422 - 0.7706989j$.

$$F_2 = \begin{bmatrix} -172.58241 & 135.085 \\ -173.19315 & 135.43792 \end{bmatrix}$$

where the eigenvalues are $\lambda_1 = -3$, $\lambda_2 = -4$, $\lambda_3 = -22.229984$, $\lambda_4 = -0.509633$, $\lambda_5 = -1.5034249$, $\lambda_6 = -5.2611072$, $\lambda_7 = -0.7730653$.

V. CONCLUSIONS

This paper has addressed the approach to partial eigenvalue assignment for the descriptor system with the condition of $S-$controllability. The impulse elimination problems via a class of feedback controllers for the second-order system using the ESA approach and Sylvester equations were addressed. Two theorems were presented using Sylvester's equations. Two algorithms were implemented based on the Sylvester equation, and examples were presented with their conclusions.

REFERENCES

[1] Y. Dumont, D. Goeleven and M. Rochdi, "Reduction of second order unilateral singular systems", Applications in mechanics, *J. Appl. Math. Mech.*, vol. 81, no. 4, pp. 219-–245, 2001.

[2] W. Marszalek and H. Unbehauen, "Second order generalized linear systems arising in analysis of flexible beams", *Proc.* 31^{st} *Conf. on Decision & Control*, Tucson, AZ, December, pp. 3514–3518, 1992.

[3] M. Hajarian, "Matrix form of the Bi-CGSTAB method for solving the coupled Sylvester matrix equations", *IET Control Theory Appl.*, vol. 7, no. 14, pp. 1828-–1833, 2013.

[4] M. Dehghan and M. Hajarian, "Efficient iterative method for solving the secondorder Sylvester matrix equation EVF2-AVF-CV=BW", *IET Control Theory Appl.*, vol.3, no. 10, pp. 1401—1408, 2009.

[5] G. R. Duan, "Parametric eigenstructure assignment in second-order descriptor linear systems", *IEEE Trans. Autom. Control*, vol. 49, no. 10, pp. 1789–1795, 2004.

[6] T. H. S. Abdelaziz, "Eigenstructure assignment for second-order systems using velocity-plus-acceleration feedback", *Struct. Control Health Monit.*, vol. 20, no. 4, pp. 465-–482, 2013.

[7] T. H. S. Abdelaziz, "Robust pole placement for second-order linear systems using velocity-plus-acceleration feedback", *IET Control Theory Appl.*, vol. 7, no. 14, pp. 1843-–1856, 2013.

[8] T. H. S. Abdelaziz, "Parametric approach for eigenstructure assignment in descriptor second-order systems via velocity-plus-acceleration feedback", *J. Dyn. Sys., Meas., Control*, vol. 36, no.4, 2014.

[9] G. R. Duan and G. P. Liu, "Complete parametric approach for eigenstructure assignment in a class of second order system", *Automatica*, vol. 38, no. 4, pp. 725—729, 2002.

[10] T. H. S. Abdelaziz and M. Valasek, "Eigenstructure assignment by proportional-plusderivative feedback for second-order linear control systems", *Kybernetika*, vol. 41, no. 5, pp. 661–676, 2005.

[11] G. R. Duan, "Analysis and design of descriptor linear systems", *Springer*, 2010.

[12] B. Zhang, "Parametric eigenstructure assignment by state feedback in descriptor systems", *IET Control Theory Appl.*, vol. 2, no. 4, pp. 303-–309, 2008.

[13] B. Zhou, Z. Y. Li and G. R. Duan, "Optimal pole assignment for discrete time systems via Stein equations", *IET Control Theory Appl.*, vol. 3, no.8, pp. 983—994, 2009.

[14] H. Ouyang, "A hybrid control approach for pole assignment to secondorder asymmetric systems", *Mech. Syst. Signal Process.*, vol.25, no. 1, pp. 123—132, 2011.

[15] J. M. Araújo, C. E.T. Dórea, L. M.G. Gonçalves and B. N. Datta, "State derivative feedback in second-order linear systems: A comparative analysis of perturbed eigenvalues under coefficient variation", *Mechanical Systems and Signal Processing*, vol. 76-77, pp. 33–46, 2016.

[16] Y. Teng and G. R. Duan, "Solvability and complete parametric approach for output regulation problems of matrix second-order systems via error feedback", *J. Harbin Inst. Technol.*, vol. 39, no. 5, pp. 691–695, 2007.

[17] X. Zhang and X. Liu, "Output regulation for matrix second order singular systems via measurement output feedback", *J. Franklin Inst.*, vol. 349, no.6, pp. 2124–2135, 2012.

[18] Y. Kim, H. S. Kim and J. L. Junkins, "Eigenstructure assignment algorithm for mechanical second-order systems", *AIAA J. Guidance Control Dyn.*, vol. 22, no. 5, pp. 729–731, 1999.

[19] D. J. Inman, "Vibration with control", *(John Wiley & Sons Ltd*, Chichester, England, 2006.

[20] N. K. Nichols and J. Kautsky, "Robust eigenstructure assignment in quadratic matrix polynomials: nonsingular case", *SIAM J. Matrix Anal. Appl.*, vol. 23, no. 1, pp. 77—102, 2001.

[21] G. R. Duan and L. Huang, "Robust pole assignment in descriptor second-order dynamical systems", *Acta Autom. Sin*, vol.33, no. 8, pp. 888—892, 2007.

[22] S. Xu and J. Qian, "Orthogonal basis selection method for robust partial eigenvalue assignment problem in second-order control systems", *J. Sound Vib.*, vol.317, no.(1–2), pp. 1—19, 2008.

[23] M. G. Tehrani and H. Ouyang, "Receptance-based partial pole assignment for asymmetric systems using state-feedback", *Shock Vib.*, vol. 19, no. 5, pp. 1135–1142, 2012.

[24] J. Zhang, H. Ouyang and Y. Zhang, "Partial quadratic eigenvalue assignment in vibrating systems using acceleration and velocity feedback", *Inverse Probl Sci. Eng.* vol. 23, no.3, pp. 479–497, 2015.

[25] J. Zhang, H. Ouyang and J. Yang, "Partial eigenstructure assignment for undamped vibration systems using acceleration and displacement feedback", *J Sound Vib.*, vol. 333, no. 1, pp. 1–12, 2014.

[26] G. S. Zhang and W. Q. Liu, "Impulsive mode elimination for descriptor systems by a structured P-D feedback", *IEEE Trans. Autom. Control*, vol. 56, no. 12, pp. 2968–2973, 2011.

[27] M. Hou, "Controllability and elimination of impulsive modes in descriptor systems", *IEEE Trans. Autom. Control*, vol. 49, no. 10, pp. 1723–1727, 2004.

[28] G. R. Duan and A. G. Wu, "Impulse elimination via state feedback in descriptor systems", *Proc. Int. Conf. Impulsive Dyn. Sys. Appl.*, Wuxi, China, October, pp. 722–729, 2005.

[29] S. Shi, Q. Zhang and Z. Yuan, "Hybrid impulsive control for switched singular systems", *IET Control Theory Appl.*, vol 5, no. 1, pp. 103–111, 2011.

[30] P. Yu and G. Zhang, "Eigenstructure assignment and impulse elimination for singular second-order system via feedback control", *IET Control Theory & Applications*, vol. 10, no. 8, pp.869–876, 2016.

[31] A. G. Wu, G. R. Duan and S. M. Zhao, "Impulsive-mode controllablisability in descriptor linear systems", *IET Control Theory Appl.*, vol. 1, no. 3, pp. 558–563, 2007.

[32] Z. B. Yan, "Disturbance impulse controllability in descriptor system", *IEEE Trans. Autom. Control*, vol. 56, no. 4, pp. 946–951, 2011.

[33] P. Losse and V. Mehrmann, "Controllability and observability of second order descriptor systems", *SIAM J. Control Optim.*, vol. 47, no. 3, pp. 1351–1379, 2008.

[34] M. Hou, "Controllability and Elimination of Impulsive Modes in Descriptor Systems" *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1723-–1727, 2004.

[35] D. Henrion, J. C. Zúñiga, "Detecting infinite zeros in polynomial matrices", *IEEE Trans. Circuits Syst. II-Express Briefs*, vol. 52, no. 12, pp. 744–745, 2005.

[36] E. B. Castelan, "Estabilização de sistemas descritores por realimentação de saídas via subespaços invariantes", *Revista Controle & Automação*, vol.16 no.4, pp. 467–477,Outubro,2005.

[37] E. B. Castelan, and V. G. Da Silva, "On the solution of a sylvester equation appearing in descriptor systems control theory", *Systems and Control Letters*, vol. 54, no.2, pp 109—117. 2005.

[38] E. R. L. Villarreal, "Abordagem Geométrica para Estabilização por Realimentação de Saídas e sua Extensão aos Sistemas Descritores", *Doutorado em Engenharia Elétrica-UFSC*, Florianopolis, SC, Brasil, 2002.

[39] V. K. Mishra, N. K. Tomar and M. K. Gupta "On Controllability and Normalizability for Linear Descriptor Systems", *Journal of Control, Automation and Electrical Systems* vol. 27, pp.19–28, 2016.

[40] J. Zhang, Y. Yuan and H. Liu, "An Approach to Partial Quadratic Eigenvalue Assignment of Damped Vibration Systems Using Static Output Feedback", *International Journal of Structural Stability and Dynamics*, vol. 18, no. 1, 1850012, pp. 1–18, 2018.