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Abstract

In the context of modern agriculture, innovative methods have emerged, including Smart Floating Farms (SFF), sustainable structures capable of producing food on sea surfaces, lakes and rivers. We focus on the middle level of the SFF, where food production takes place. We present a Mixed Linear Integer Programming model to solve the Two-Dimensional Strip Packing Problem, taking into account different size options for plots of land each crop species, with the objective of minimizing the height at the middle level. To validate our model we using the MINIZINC constraint modeling language on a set of randomly generated instances.

1 Introduction

Nowadays, climate change is an undeniable and worrisome event throughout the world caused mainly by the irrational actions of the human being in nature, such as so-called industrial agriculture that focuses on the mass production of food. Smart Floating Farms (SFF)[1]) are innovative methods of modern agriculture. They are structures that are sustainable for contributing to making food production more transparent, since they promote the use of renewable energy to produce foods from scratch km [1, 2]. These are composed of a three-tier system: a. solar energy (*rooftop*), b. hydroponics (*middle level*) and c. fish farming (*lower level*), whose main objective is the production of food on sea surfaces, lakes or rivers [3, 4]. We will focus on the average level of these types of structures, where a set of small rectangles (parcels of land) with different size options should be adjusted as efficiently as possible.

The Two-Dimensional Rectangular Packing problem is NP-completeness [5], since it includes bin-packing as a proper case. A similar situation consists in, fitting a set $\mathcal{R}$ of N rectangles into a minimum number of large rectangles called $Strip$, without each of the small rectangles being able to overlap with the others. As an extension, the Two-Dimensional Strip Packing Problem (2DSPP) emerges, where input is known as a set $\mathcal{R} = \{r_1, \ldots, r_N\}$, the width W (finite) and height H (infinite) of the $Strip$. We will approach this extension, taking into account that there are different size options for plots of land, based on the crop marking techniques [6] used to arrange the plants within the plot of land, which determines $\approx 4\sqrt{\rho}$ for different sizes of plots for the same type of crop, where ρ is the number of plants to be placed inside the plot of land.

There are many operations research techniques to model 2DSPP as well as solving it efficiently. We propose a model of Mixed Linear Integral Programming (MILP) to solve a modified 2DSPP, taking into account different size options for plots of land of each crop species, with the objective of minimizing H.

2 2DSPP for different size options for plots of land

One of the main objectives of the SFF is to supply food adjacent areas, thus reducing their transport and inferring a decrease in polluting gases. We assume that we know for each crop species that is produced,

[1] http://smartfloatingfarms.com

the minimum average consumption of the areas that will be supplied by the SFF, which can be interpreted as the approximate number of plants ρ of a crop species that are necessary to supply the corresponding area. Each species of plant has its specification in terms of the distance that it should be within the crop rows, as well as the separation that there should be between each pair of rows, which implies, that a different arrangement of the plants in the plots of land occur every time in a different area where at least the amount of plants required can be cultivated. We then have a set of groups $\mathcal{G} = \{g_1, \ldots, g_N\}$ where each g_i represents a set of rectangles defined by the width and length of each possible plot of land of the crop species i, of which only one of them can be located at the middle level $Strip$ of the SFF. When a plot of land is selected for each crop species, the 2DSPP is solved, which consists in placing within a rectangle ($Strip$) with width $W \in \mathbb{N}$ defined, a set $\mathcal{R} = \{r_1, \ldots, r_N\}$ of N small rectangles, where $\forall r_i \in \mathcal{R}$, there $\exists w_i, h_i \in \mathbb{N}$, in such a way that there is no overlap between each pair of them [5] and searching to occupy the smallest possible height. Therefore, we take into account that between each pair of plots of land and with respect to the borders of the $Strip$, there must be at least *k* cm of distance and our objective is to select for each crop species the plot of land size, which when being arranged next to the rest of the selected ones represent the smallest space occupied by the SFF (see Figure 1), which will depend only on the height, because the width is a known parameter that is not variable.

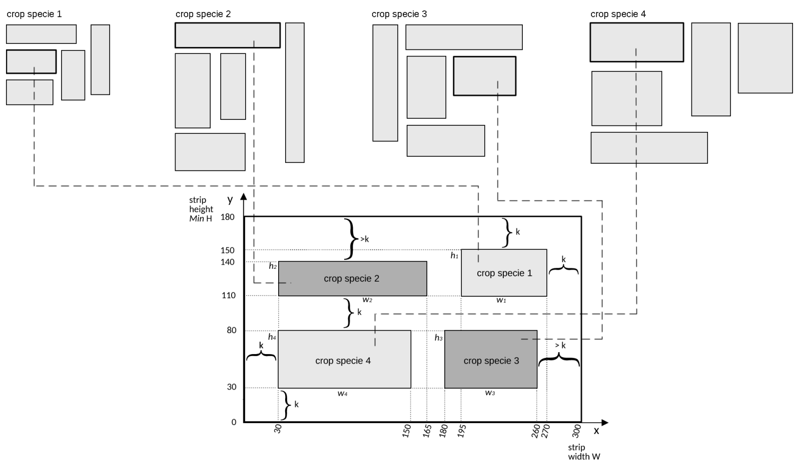

Figure 1: Process of selection and location of plots of land within the $Strip$ according to the constraints.

3 Mixed-integer linear programming model

We present a model of Mixed Integer Linear Programming (MILP) for this problem, where we consider as preprocessing of the information the determination of the width and height of the possible plots of land for each species given the number of plants ρ that are required.

- *Parameters*

$$\begin{aligned}
W &: Strip\text{ width}\\
N &: \text{Number of crop species}\\
M_x &: \text{Upper bound for } x\\
M_y &: \text{Upper bound for } y\\
C_i &: \text{Number of plots land size options for crops specie } i\\
A_{i-j} &: \text{Width of the area occupied by crop species } i\text{, if plot of land } j \text{ is selected}\\
L_{i-j} &: \text{Height of the area occupied by crop species } i\text{, if plot of land } j \text{ is selected}\\
k &: \text{Distance between each pair of plots of land and separation of the borders of the } Strip
\end{aligned}$$

- *Decision variables*

w_i : Width of the plot of land occupied by crop species i
h_i : Height of the plot of land occupied by crop species i
x_i : Coordinate x of the lower left corner of the plot of land occupied by crop species i
y_i : Coordinate y of the lower left corner of the plot of land occupied by crop species i
H : Maximum height of the $Strip$

$$Z_{i-j} = \begin{cases} 1 & \text{If for the crop species } i \text{ the land plot } j \text{ is occupied} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{i-k} = \begin{cases} 1 & \text{If the crop species } i \text{ is on the left of the crop species } k \\ 0 & \text{otherwise} \end{cases}$$

$$J_{i-k} = \begin{cases} 1 & \text{If the crop species } i \text{ is on the right of the crop species } k \\ 0 & \text{otherwise} \end{cases}$$

$$D_{i-k} = \begin{cases} 1 & \text{If the crop species } i \text{ is before (vertically) the crop species } k \\ 0 & \text{otherwise} \end{cases}$$

$$E_{i-k} = \begin{cases} 1 & \text{If the crop species } i \text{ is after (vertically) the crop species } k \\ 0 & \text{otherwise} \end{cases}$$

- *Objective*

$$Min\, Z = H \tag{1}$$

- *Constraints*

Only one plot of land option can be placed for each crop species:

$$\sum_{j=1}^{C_i} Z_{i-j} = 1 \ , \forall i = 1 \ldots N \tag{2}$$

The width of the plot of land occupied for each species will be determined by the selected option:

$$w_i = \sum_{j=1}^{C_i} A_{i-j} \times Z_{i-j} \ , \forall i = 1 \ldots N \tag{3}$$

The height of the plot of land occupied for each species will be determined by the selected option:

$$h_i = \sum_{j=1}^{C_i} L_{i-j} \times Z_{i-j} \ , \forall i = 1 \ldots N \tag{4}$$

The plots of land cannot overlap:

$$x_i + w_i + k \leq x_k + (1 - I_{i-k}) \times M_x \ , \forall i, k = 1 \ldots N, i < k \tag{5}$$

$$x_k + w_k + k \leq x_i + (1 - J_{i-k}) \times M_x \ , \forall i, k = 1 \ldots N, i < k \tag{6}$$

$$y_i + h_i + k \leq y_k + (1 - D_{i-k}) \times M_y \ , \forall i, k = 1 \ldots N, i < k \tag{7}$$

$$y_k + h_k + k \leq y_i + (1 - E_{i-k}) \times M_y \ , \forall i, k = 1 \ldots N, i < k \tag{8}$$

Each plot of land must be totally within the width of the $Strip$:

$$x_i + w_i \leq W - k \ , \forall i = 1 \ldots N \tag{9}$$

There must be at least one separation k between each plot of land and the borders of the $Strip$:

$$x_i \geq k \ , \forall i = 1 \ldots N \tag{10}$$

$$y_i \geq k \ , \forall y = 1 \ldots N \tag{11}$$

Maximum height reached within the $Strip$:

$$y_i \geq k \ , \forall i = 1 \ldots N \tag{12}$$

$$y_i + h_i \leq H \ , \forall i = 1 \ldots N \tag{13}$$

4 Implementation and Results

This model was implemented in MINIZINC [7] for macOS Sierra system. The runs were on Intel Core i5-2400 CPU 2.50GHz and 12GB of RAM. We tested our model with a set of 12 instances called RANDOMLAND, generated for this problem. Each set is made up of three types of instances: *small*, *medium* and *large*, where the numbers of plots of land are between 1 and 100 and the numbers of plants are between 1 and 300. The distance between plants and rows on a plot of land is generated for each types of instances in a range of 30 cm to 100 cm, and W is between 1000 cm and 4500 cm. The proposed optimization model provides an adequate packaging of plots of land at the middle level of the SFF and is interesting when discussing food production on smart farms and considering terms such as sustainability.

Problem data				MILP	
Instance.	#Species/Groups	Strip dimension	k	#Variables	Z_{opt}
rnd-1	10	1869	52	625	521
rnd-2	11	1878	55	749	605
rnd-3	11	1974	56	741	506
rnd-4	15	2420	58	1249	680^a
rnd-5	22	3220	59	2501	3216^a
rnd-6	22	3133	50	2521	2810^a
rnd-7	26	3362	53	3393	3753^a
rnd-8	27	3243	52	3613	4476^a
rnd-9	32	4124	57	5029	5007^a
rnd-10	33	4363	51	5309	4437^a
rnd-11	34	4108	58	5641	5426^a
rnd-12	35	3856	59	5925	6585^a

Table 1: Results for the RANDOMLAND set. Column 5 represent the quantity of decision variables and column 6 represent the values of the objective function obtained up to 10 minutes.

The Table 1 shows for the three initial instances classified as small, their optimal value found is presented, and for others (a), the best solution reached at the end of 10 minutes. Therefore, the quantities of variables for each instance classification are show in the table. It is evident that the complexity of solving the problem increases rapidly, so that for the last instance small, all medium and large, no optimal solutions were found.

5 Conclusions and Future Work

In this work we have proposed a MILP model for the 2DSPP with different size options for plots of land, and for our approach the combinatorial level of the problem scales quickly because of the number of possible plots of land for each crop of ρ plants is $\approx 4\sqrt{\rho}$, implying that the total number of possible plots of land to be combined for N crop species is $\sum_{i=1}^{N} \approx 4\sqrt{\rho_i}$, and the number of combinations for packing within the *Strip* are $\prod_{i=1}^{N} \approx 4\sqrt{\rho_i}$. We validate our model using the MINIZINC constraint modeling language and our approach is able to offer good solutions, by providing an adequate assignment for all plots of land. As future work, we will focus on adding to the proposed model a set of constraints regarding the behavior and maintenance needs of the different crops that will be packed in the SFF, and design and implement a metaheuristic algorithm to cover this problem and get better results.

6 Acknowledgments

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