Coproducts of Distributive Lattice based Algebras

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The analysis of coproducts in varieties of algebras has generally been variety-specific, relying on tools tailored to particular classes of algebras. A recurring theme, however, is the use of a categorical duality. Among the dualities and topological representations in the literature, natural dualities are particularly well behaved with respect to coproduct. Since (multisorted) natural dualities are based on hom-functors, they send coproducts into cartesian products.

We carry out a systematic study of coproducts for finitely generated quasivarieties $\mathcal{A}$ that admit a (term) reduct in the variety $\mathcal{D}$ of bounded distributive lattices. In this setting we present necessary and sufficient conditions on $\mathcal{A}$ for the forgetful functor $\mathsf{U}_{\mathcal{A}}$ from $\mathcal{A}$ to $\mathcal{D}$ to preserve coproducts. When this is not the case, we demostrat how to obtain the lattice reduct of a coproduct of algebras in $\mathcal{A}$ from the lattice reducts of the algebras involved.

Our results are based on an in depth analysis of the connection between Priestley duality and multisorted piggyback dualities. Given an algebra $A \in \mathcal{A}$ we present procedure to recover the Priestley dual of $\mathsf{U}_{\mathcal{A}}(A)$ from the natural dual of A. We then use this translation and the good behaviour of natural dualities with coproducts to recover distributive lattice reduct of a coproduct of algebras in $\mathcal{A}$.

As a byproduct, our work reveals that the type of natural duality that the class $\mathcal{A}$ can possess is connected with properties of coproducts in $\mathcal{A}$ and the way in which $\mathsf{U}_{\mathcal{A}}$ behaves with respect to them.

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