EPiC Series in Computing
Volume 98, 2024, Pages 179–188
Proceedings of 39th International Conference on Computers and Their Applications

Bike sharing systems via birth-death process and simulation modelling

Lena Wallwork, Uchechukwu Obinwanne, and Wenying Feng*

Department of Computer Science
Trent University Durham Greater Toronto Area
Oshawa, Ontario, Canada, L1J 5Y1
{lenawallwork, uchechukwuobinwanne, wfeng}@trentu.ca

Abstract

In this paper, we present a simulation study for a bike-sharing network. The model is analyzed with the Birth-Death process as well as a multidimensional Markov queueing system. We evaluate the steady-state probability of running out of docking stations or bikes. The results provide guidelines for the setup of the network with optimized system efficiency.
Keywords: Bike sharing, birth-death process, equilibrium state, Markov network.

1 Introduction

In today's advanced world, resource-sharing concepts such as vehicle-sharing, or bike-sharing systems are becoming increasingly popular. Bike sharing began in the 1960s in Amsterdam and spread to different parts of the world [6]. Bike-sharing services are perceived as a great means of transportation for short trips because they are more environment-friendly, involve fewer headaches for tasks such as finding parking, getting insurance, performing maintenance, and require less overhead investment from customers.

As an interdisciplinary research area, bike-sharing systems have been widely studied from different perspectives including management [10], transportation [3], geography [1], medicine [4], environmental science [11, 12], sustainability [2], mathematics [5], big data [12] and computer science [9]. It has been shown that bike-sharing programs have a positive impact on the consumption of natural resources, air pollution, intelligent transportation systems, health and the economy. It helps to promote social and leisure activities, increase transport choice and convenience, and reduce travel time and cost. On the other side, it is also shown that it does not reduce traffic congestion and carbon emissions. A recent review of impact of bike-sharing programs for smart cities is given in [2].

One of the major challenges encountered in bike-sharing systems is the restocking of the bikes to the docking stations in a way that the system is optimized by the measure of customers

*Corresponding author.

satisfaction and resource management. In this paper, we use a computer simulation technique to study the restocking of a bike-sharing system aimed at an efficient performance from both the points of management and customers. In addition, a network model following the Birth-Death (BD) system, and a multidimensional Markov process are presented to analyze the performance of the system. Steady-state probabilities of running out of docking stations or bikes are obtained by the combination of simulation and Markov models. The results provide insight into a well-balanced system and suggestions for the implementation of management policies.

The rest of the paper is organized as follows. In Section 2, we present birth-death models with some equilibrium analysis. Experiments and performance comparisons are discussed in Section 3. Lastly, some conclusions and recommendations are summarized in Section 4.

2 Birth-death process for bike sharing system modelling

As an example, the network of a 4-station system is shown in Figure 1. For a given station, we refer the definitions and terminologies listed in Table 1. Customers arrive at a station to rent a bike for a duration of time and use it to make a trip from the original station to a destination. The difference in customer arrival rates at each station can result in an imbalanced distribution of bikes among the bike stations. This may result in two undesirable scenarios: 1- the station is full so returning a bike is not possible, and 2- customers cannot rent a bike due to no availability. In either of these scenarios, the customer has to exit the system immediately and try another time. An alternative assumption is to force the customers to wait till availability arises.

Original Station	Where a bike is rented from
Destination Station	Where a bike is returned to
Trip Start Time	When a bike is checked out
Trip End Time	When a bike is returned to a docking station
Trip duration	The difference between Trip Start Time and Trip End Time
Rate of bike check-out (μ)	Rate of bikes being checked out at a station
Rate of bike return (λ)	Rate of bikes being returned to a station
Transfer rate	Rate of bikes transferring from one station to another
Station capacity	Total number of docks in the station

Table 1: Definitions

The following birth-death models are based on a single station. The state of the station is defined to be the number of bikes available at the station.

Case 1: negative states are not considered

Assume that the transfer rates are independent of time so that we can model each station as an $M/M/1/K$ queue. To be able to solve this system as a Markov process, the system's state needs to be defined in a way that the future state only depends on that of the present and not that of the past. We define our system's states based on the number of available bikes at the stations. Thus the array of states is shown in (1). This case assumes that if a customer arrives at a troubled station (no bikes or docks), he or she will have to exit the system immediately.

$$X = [0, 1, ..., K-1, K], \quad \text{where K is the capacity of the station.} \tag{1}$$

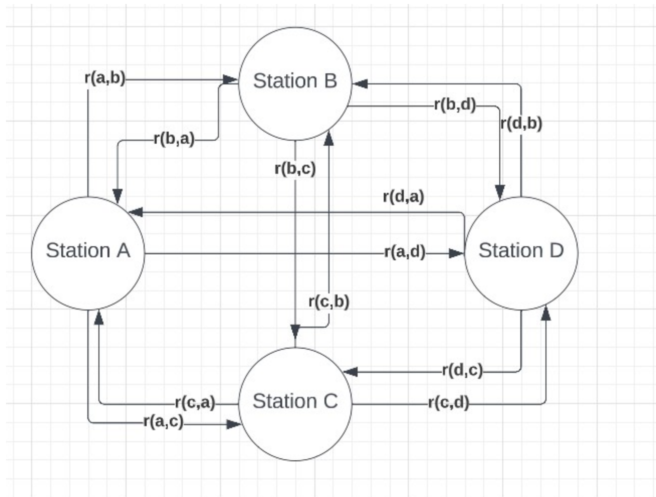


Figure 1: Network of stations

The state transition diagram is simply shown in Fig. 2, where $\mu_i,\ (i = 1, 2, \cdots, K)$ are the rates of checking out bikes, and $\lambda_i,\ (i = 0, 1, 2, \cdots, K-1)$ are the rates of returning bikes at a specific station.

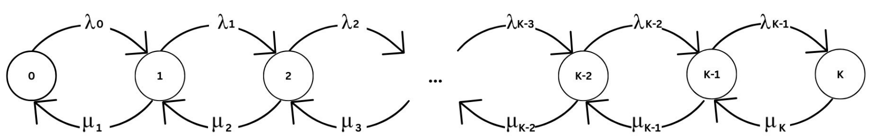


Figure 2: Case 1: State transition diagram

Assuming constant rates for both checking out and returning, the probability of zero customers in the state can be calculated by equilibrium analysis for the birth-death process:

$$P_0 = \frac{1-\rho}{1-\rho^{k+1}}, \text{ where } \rho = \frac{\lambda}{\mu} < 1. \tag{2}$$

Case 2: negative states are considered

In case 2, we assume that customers arriving at a troubled station will wait till a dock/bike becomes available. Let the number of docks at station A be K, the maximum number of customers waiting for a bike be N, and the maximum number of customers waiting for a dock be M. To define the state parameter, the following two scenarios are considered:

(1) The number of bikes at Station A is non-zero, and the number of customers waiting for docks is zero or positive;

(2) The number of bikes at Station A is zero, and there are customers at the station waiting for a bike.

In the first scenario, the state parameter is defined as the sum of the number of bikes in the system and the number of customers waiting for a dock to return a bike. If the number of bikes is less than K, then $M = 0$. The state parameter is non-positive. In this second scenario, the state parameter is defined to be negative with the absolute value as the number of customers waiting for a bike. The system's total number of states is $2M + N - 1$ as listed below.

$$X = [-N, -N+1, \cdots, -1, 0, 1, 2, \cdots, 2M-1, 2M]. \tag{3}$$

Fig. 3 shows the state transition diagram for the special case $M = N = K$. It can be easily generated to the general case with M, N and K are any positive integers.

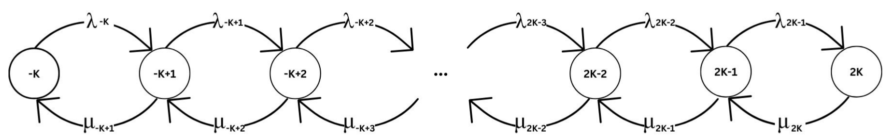
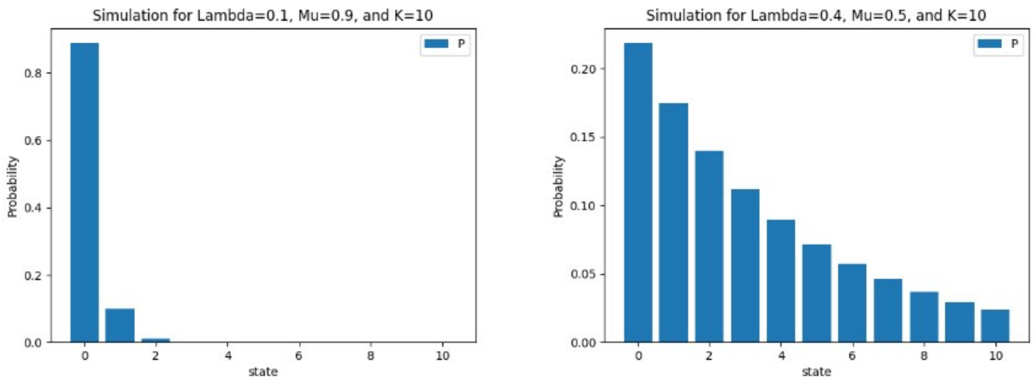


Figure 3: Case 2 state transition diagram

Similar to case 1, assuming constant rates for both checking out and returning, the probability of zero customers in the state can be calculated. To be complete, we show the balance equations for the equilibrium states:

$$\text{State } -K: \lambda P_{-K} = \mu P_{-K+1}$$
$$\rightarrow \quad P_{-K+1} = \frac{\lambda}{\mu} P_{-K} \tag{4}$$
$$\text{State } -K+1: \lambda P_{-K} + \mu P_{-K+2} = (\lambda + \mu) P_{-K+1}$$
$$\rightarrow \quad P_{-K+2} = \left(\left(\frac{\lambda}{\mu}\right)^2 + \frac{\lambda}{\mu} \right) P_{-K} - \frac{\lambda}{\mu} P_{-K} = \left(\frac{\lambda}{\mu}\right)^2 P_{-K} \tag{5}$$
$$\text{State } 2K-2: \lambda P_{2K-3} + \mu P_{2K-1} = (\lambda + \mu) P_{2K-2}$$
$$\rightarrow \quad P_{-K-1} = \left(\left(\frac{\lambda}{\mu}\right)^{K-1} + \frac{\lambda}{\mu} \right) P_{-K} - \frac{\lambda}{\mu} P_{-K} \quad = \left(\frac{\lambda}{\mu}\right)^{3K-1} P_{-K} \tag{6}$$
$$\text{State } 2K-1: \lambda P_{2K-2} + \mu P_{2K} = (\lambda + \mu) P_{2K-1}$$
$$\rightarrow \quad P_{2K} = \left(\left(\frac{\lambda}{\mu}\right)^{K} + \frac{\lambda}{\mu} \right) P_{-K} - \frac{\lambda}{\mu} P_{-K} = \left(\frac{\lambda}{\mu}\right)^{3K} P_{-K} \tag{7}$$
$$\text{State } 2K: \lambda P_{2K-1} = \mu P_{2K}$$
$$\rightarrow \quad P_{2K} = \left(\left(\frac{\lambda}{\mu}\right)^{K} + \left(\frac{\lambda}{\mu}\right)^2 \right) P_{-K} - \left(\frac{\lambda}{\mu}\right)^2 P_{-K} = \left(\frac{\lambda}{\mu}\right)^{3K} P_{-K} \tag{8}$$

$$\text{Using the fact that} \quad \sum P_i = 1 \quad \text{for } -K \le i \le 2K$$
$$\text{we have} \sum \left(\frac{\lambda}{\mu}\right)^{i+K} P_0 = 1 \quad \text{for } -K \le i \le 2K$$



(a) Case 1: $\rho = \frac{1}{9}$

(b) Case 1: $\rho = \frac{4}{5}$

Figure 4: Case 1: Effects of ρ on the probability distribution

Therefore, $P_0 = \frac{1-\rho}{1-\rho^{3K+1}}$, where $\lambda_i = \lambda$ $(i = -K, -K = 1, \cdots, 2K-1)$, $\mu_i = \mu$ $(i = -K+1, -K = 2, \cdots, 2K)$ and $\rho = \frac{\lambda}{\mu} < 1$. Our experiments presented next section follow the process shown as Fig. 3 for case 2.

3 Experiments and comparison

3.1 Steady-state simulation - case 1

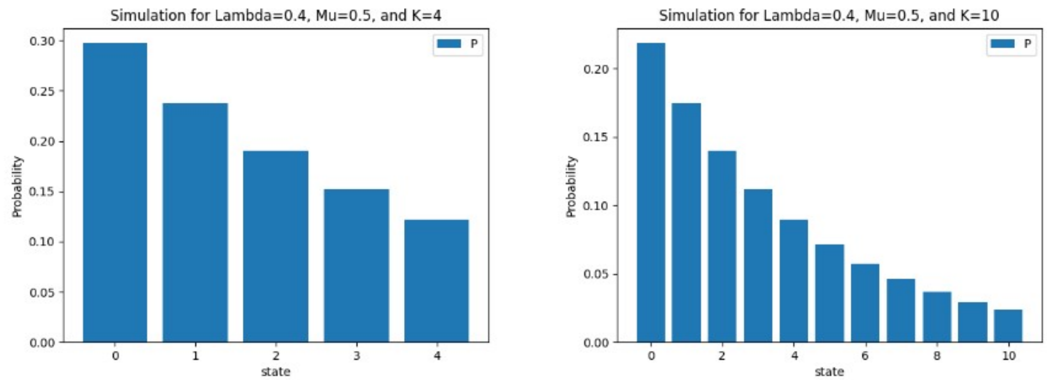
Various simulations in Python have been performed to study the effect of different parameters on the probability of having a troubled station (probability of state 0 or K). First the effect of ρ on the steady-state probability distribution of each state for an individual station is studied. We have calculated the steady-state probability distribution of all possible states for low, medium, and high values of ρ. Fig. 4 summarizes the results from the experiments.

As expected, a high ratio of ρ will push the system towards maxing out the capacity and available docks at the station, vs. a low ratio of ρ which will result in the unavailability of bikes for customers. An ideal situation is a case where $\rho = 1$, that is, for every bike checkout, there is a corresponding bike return.

Next, the effect of capacity on probability distribution is displayed in Fig. 5. This effect on P_0 is observed more precisely in Fig. 6. It is visible in the figure that an increase in capacity results in a decrease in the probability of state 0, which is equivalent to having no available bikes for customers.

Next, the average number of bikes/docks in the system is calculated. Fig. 7 illustrates the effect of the ratio of λ over μ on the average number of bikes in this system. As can be seen in this figure, as λ over μ ratio increases, the number of available bikes increases, while available docks drop.

Consequently, the average number of failed returns (when no docks are available) increases while the number of failed checkouts (when no bikes are available) drops to zero as seen in Fig. 8.



(a) Case 1: $\rho = \frac{4}{5}$ and $K = 4$

(b) Case 1: $\rho = \frac{4}{5}$ and $K = 10$

Figure 5: Case 1: Effects ρ and K on the probability distribution

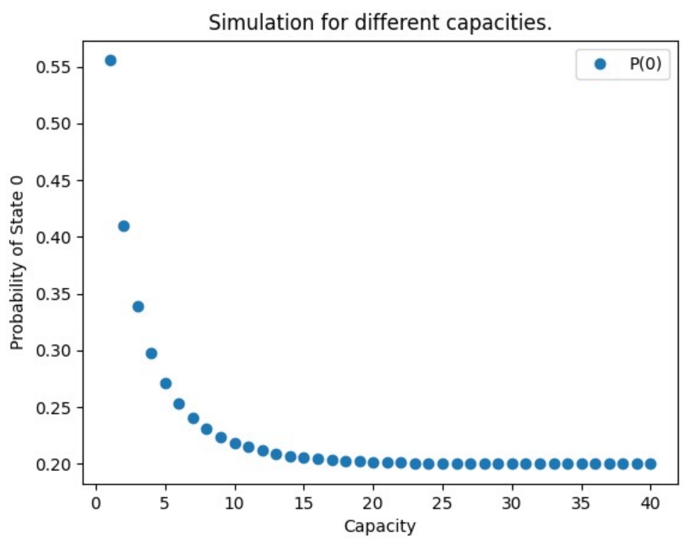


Figure 6: Case 1: Effect of capacity on the probability of the first (troubled) state

3.2 Steady-state simulation - case 2

First the effect of ρ on the steady-state probability distribution of each state in Fig. 9. Similarly to Fig. 4, it is visible that as the ratio of bikes returned to bikes checked out increases, the lower the probability of having docks with no available bikes (state 0).

The effect of capacity on this probability distribution is seen in Fig. 10. The higher the capacity of the station, the greater the likelihood of bike unavailability and consequently, more customers waiting.

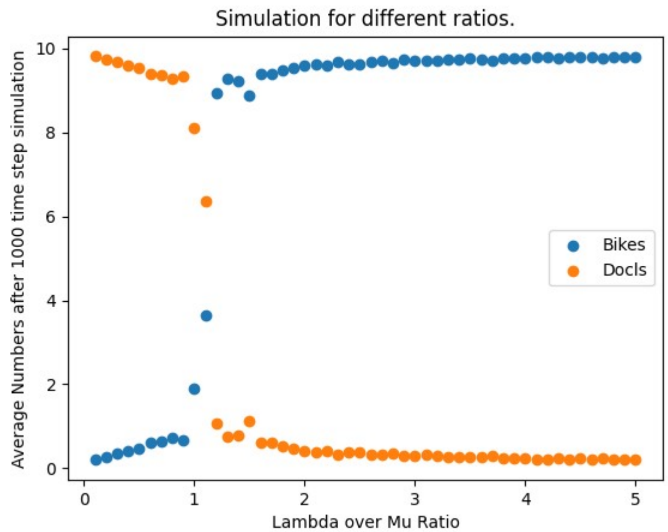


Figure 7: Case 2: Effect of ρ

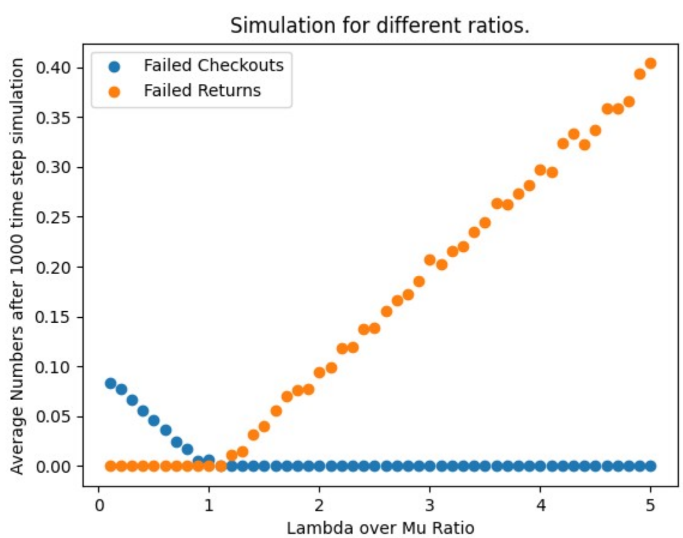
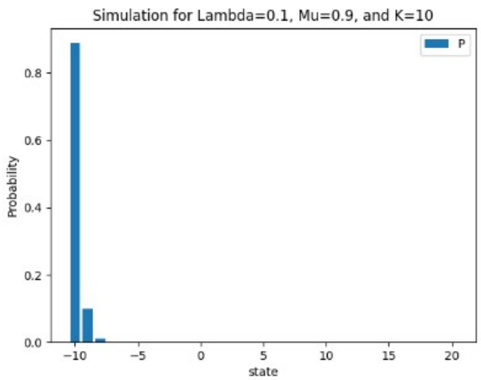
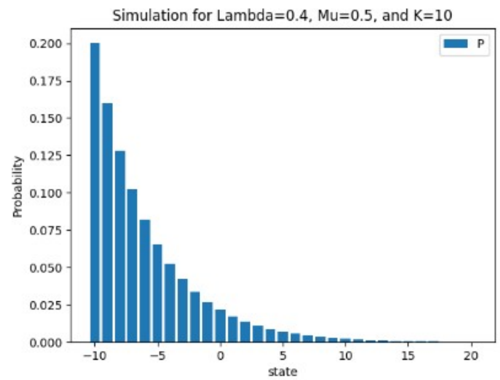


Figure 8: Case 2: effect of ρ

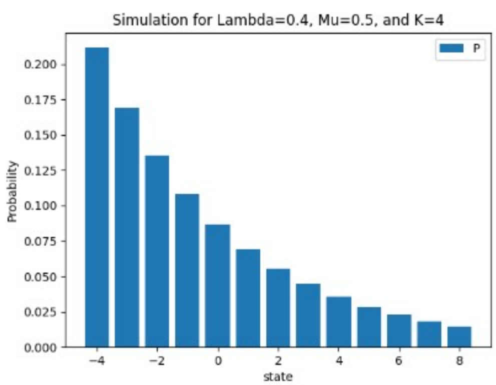


(a) Case 2: $\rho = \frac{1}{9}$

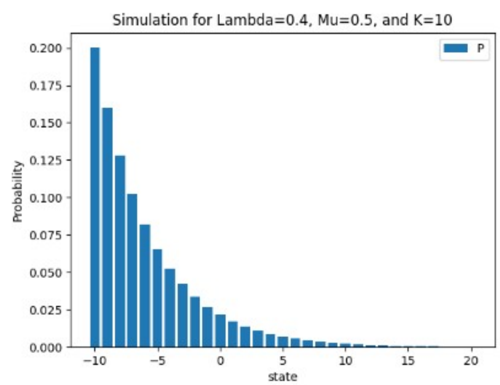


(b) Case 2: $\rho = \frac{4}{5}$

Figure 9: Case 2 probability distribution for different values of ρ



(a) Case 2: $\rho = \frac{4}{5}$ and $K = 4$



(b) Case 2: $\rho = \frac{4}{5}$ and $K = 10$

Figure 10: Case 2: effect of ρ and K

The effect of capacity on P_{-K}, where troubled states have lower probabilities vs. case 1 is shown in Fig. 11. As the capacity increases, the probability of state $-K$ decreases.

4 Conclusions and recommendations

In this paper, a bike-sharing station with two resources (bikes and docks) and two types of services (bike returns or bike check-outs) has been modelled as an $M/M/1/K$ queue. Two cases have been simulated and compared under two different scenarios. In case 1, customers exit the system in case of no availability. The state variable here is the number of available bikes at the station which can range between 0 and $K + 1$, where K is the capacity of the station. In case 2, up to K customers can wait to check out or return a bike. Here the state

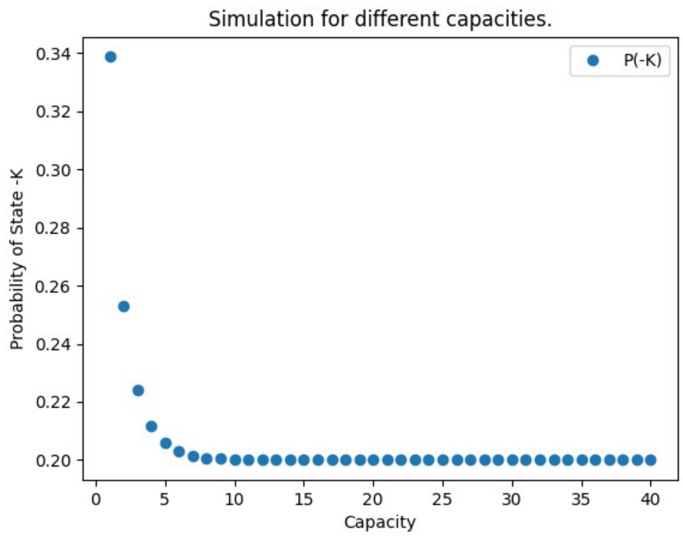


Figure 11: Case 2: Effect of capacity on the probability of the first (troubled) state

variable extends to negative values and can fall between $-K$ and $2K$, to account for customers waiting for either of the services.

First, the effect of ρ, the ratio of return rates to checkout rates on the steady-state probability distribution of each state has been studied. The simulation illustrates that higher ratios of ρ will result in a higher probability of having more bikes available for case 1 and having K customers waiting for bikes in case 2.

Next, the effect of the capacity of the station on the probability of the troubled state of 0 bikes or K customer waiting has been studied. The simulation results show that increasing the capacity of the station drops this probability, and ultimately for the case of $\rho < 1$ this probability converges to 0.2 where further increase in capacity will not result in any additional improvement.

Finally, comparing case 1 to case 2, it can be concluded that forming a waiting line-up for customers can reduce the probability of having a troubled state, depending on the capacity of the station. In stations with lower capacity, this effect is more evident.

In future works, a network of stations can be modelled to study the interrelationship between stations. In this scenario, each station can also be modeled with the assumption that when there are no bikes or docks available, customers will transit to a nearby station.

References

[1] L. Cheng, Z. Mi, D. Coffman, J. Meng, D. Liu and D. Chang, The role of bike sharing in promoting transport resilience, Netw. Spat Econ **22**, pp. 567–585 (2022). https://doi.org/10.1007/s11067-021-09518-9.

[2] E. Eren and V. E. Uz, A review on bike-sharing: The factors affecting bike-sharing demand, Sustainable Cities and Society **54**, (2020), 101882, ISSN 2210-6707. https://doi.org/10.1016/j.scs.2019.101882.

[3] E. Fishman, S. Washington and N. Haworth, Barriers and facilitators to public bicycle scheme use: A qualitative approach, Transportation Research Part F: Traffic Psychology and Behaviour **15**, Issue 6, (2012), pp. 686-698, ISSN 1369-8478. https://doi.org/10.1016/j.trf.2012.08.002.

[4] D. Fuller, L. Gauvin, Y. Kestens, M. Daniel, M. Fournier, P. Morency and L. Drouin, Use of a new public bicycle share program in Montreal, Canada, American Journal of Preventive Medicine **41**, Issue 1, (2011), pp. 80-83, ISSN 0749-3797. https://doi.org/10.1016/j.amepre.2011.03.002.

[5] J. R. Graef, S. Ho, L. Kong and M. Wang, A fractional differential equation model for bike share systems, Journal of Nonlinear Functional Analysis **2019**, (2019), pp. 1-14, Article ID 23. https://doi.org/10.23952/jnfa.2019.23.

[6] Q. Li, C. Chen, R. Fan, L. Xu and J. Ma, Queueing analysis of a large-scale bike sharing system through mean-field theory, (2016). https://doi.org/10.13140/RG.2.1.4023.0800.

[7] F. Leurent, Modelling a vehicle-sharing station as a dual waiting system: stochastic framework and stationary analysis, (2012), hal-00757228. https://hal.science/hal-00757228.

[8] C. Moon, A. B. Sharpin, D. L. L. Iván, K. Azra, L. R. Luca and A. Maassen, The evolution of bike sharing: 10 questions on the emergence of new technologies, opportunities, and risks, (2019). https://www.wri.org/research/evolution-bike-sharinge.

[9] S. Man, R. Zhou, B. Kam and W. Feng, Prediction of bike sharing activities using machine learning and data analytics, ITNG 2023 20th International Conference on Information Technology-New Generations, Advances in Intelligent Systems and Computing **1445**, (2023), ISBN 978-3-031-28332-1. https://www.springerprofessional.de/en/prediction-of-bike-sharing-activities-using-machine-learning-and/25328454.

[10] M. Ricci, Bike sharing: A review of evidence on impacts and processes of implementation and operation, Research in Transportation Business and Management **15**, (2015), pp. 28-38, ISSN 2210-5395. https://doi.org/10.1016/j.rtbm.2015.03.003.

[11] T. D. Trana, N. Ovtrachta and B. F. d'Arciera, Modeling bike sharing system using built environment factors, Procedia CIRP **30**, (2015), pp. 293-298, ISSN 2212-8271. https://doi.org/10.1016/j.procir.2015.02.156.

[12] Y. Zhang and Z. Mi, Environmental benefits of bike sharing: A big data-based analysis, Applied Energy **220**, (2018), pp. 296-301, ISSN 0306-2619. https://doi.org/10.1016/j.apenergy.2018.03.101.