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Identification of the Aquifer Parameters from Pumping Test Data by using a Hybrid Optimization Approach

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Abstract

The main objective of this study is to propose a linked simulation-optimization approach to determine the parameters of the confined and leaky-confined aquifers from the results of the pumping tests. In the simulation part of the proposed approach, the drawdowns at the given monitoring points and times are calculated by considering Theis and Hantush approaches for confined and leaky-confined aquifers, respectively. This simulation part is then integrated with a hybrid optimization approach where global exploration feature of the harmony search (HS) and strong local search capability of the generalized reduced gradient (GRG) approach of the spreadsheet Solver add-in are mutually integrated. The performance of the proposed approach is evaluated by considering two pumping test data for the confined and leaky-confined aquifers. Identified results indicated that the hybrid HS-Solver optimization approach provides better results than those obtained by using both curve matching and stand-alone HS approaches.

1 Introduction

Groundwater management is an important part of the water resources planning and management. Mathematical simulation models are the essential tools for developing sustainable management plans for groundwater resources which require of knowing the spatial distributions of aquifer parameters over the field. These parameter distributions are usually obtained by interpolating the point observations which are mostly determined by performing pumping tests. Note that most of these tests are conducted by extracting the groundwater with a constant rate and recording the corresponding drawdowns in different monitoring locations and times. After obtaining the time-drawdown data from pumping test results, the parameters of the aquifer system are traditionally determined by means of a manual curve

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matching approaches. Although these approaches are simple to employ, some errors might be introduced since the accuracy of these approaches is mostly dependent to the modeller's ability (Huang and Yeh, 2008). Therefore, use of the simulation-optimization models becomes popular tools to determine the aquifer parameters.

The current literature includes various solution approaches that integrate different simulation and optimization models for determining the aquifer parameters. These models mainly differ in terms of the considered optimization approaches. Both deterministic and heuristic optimization approaches are used employed to the solution of aquifer parameter estimation problems. Although deterministic approaches such as conjugate gradient, Gauss-Newton, and Marquardt algorithms are efficient to solve the problem in reasonable times, they usually prone to finding the local optimum solutions if a special initial solution is not provided (Samuel and Jha, 2003). The reason of this problem is associated with the non-convex solution space of the groundwater optimization problems (Willis and Yeh, 1987; Sun, 1994). Consequently, heuristic optimization approaches such as simulated annealing (SA) (Huang and Yeh, 2008), genetic algorithm (GA) (Samuel and Jha, 2003), and particle swarm optimization (PSO) (Sahin, 2018) algorithms are preferred to use in the solution of aquifer parameter estimation problems. Note that heuristic approaches are usually inspired from some events in natural phenomena and usually find the global or near global optimum solutions no matter where the solution starts (Ayvaz et al., 2009). However, stand-alone use of these approaches may require long computation times to precisely find the global optimum solutions (Michalewicz, 1992). Therefore, hybrid optimization approaches are employed for solving the optimization problems with a non-convex solution space. The main philosophy of the hybrid optimization approaches is to integrate the global exploration feature of the heuristic approaches and strong local search ability of the deterministic optimization approaches. In these algorithms, the global exploration process starts with multiple starting points and explores the search space, and then, deterministic approaches find the optimum solution by taking the results of global exploration as their initial values (Ayvaz et al., 2009). This kind of an integration makes the solution of the problem more robust than both heuristic and deterministic optimization approaches by themselves (Shannon, 1998).

The main objective of this study is to propose a hybrid solution approach for solving the aquifer parameter estimation problems by using the pumping test data. In the proposed approach, drawdown values at given distance and times are calculated by using the Theis and Hantush equations for confined and leaky-confined aquifers, respectively. These solutions are then integrated to an optimization model where hybrid HS-Solver optimization approach is used. HS-Solver is a recently proposed hybrid optimization approach which integrates the heuristic harmony search (HS) algorithm and a generalized reduced gradient (GRG) approach in the spreadsheet Solver add-in as the global and local optimizers, respectively. The performance of the proposed hybrid approach is evaluated by considering two pumping test data for the confined and leaky-confined aquifers. Identified results indicated that the proposed hybrid approach provides better results than those obtained by using both curve matching and stand-alone HS approaches.

2 Model Development

In this section, the main structure of the proposed hybrid approach is presented. For this purpose, first, the mathematical formulations of the Theis and Hantush models are given, and then, how to integrate them to the hybrid HS-Solver optimization approach is presented.

2.1 Theis Model

Unsteady groundwater flow toward a fully penetrating well in a confined aquifer is represented by Theis model which is expressed as follows (Theis, 1935):

$$s(r,t) = \frac{Q}{4\pi T} W(u) \tag{1}$$

where $s(r,t)$ is the drawdown at a distance r and at a time t, Q is the pumping rate which is constant during test, T is the transmissivity of the aquifer, u is a dimensionless parameter, and $W(u)$ is the well function of u, which is also known as the Theis function. Dimensionless parameter of u and well function of $W(u)$ are given as follows:

$$u = \frac{r^2 S}{4Tt} \tag{2}$$

$$W(u) = \int_u^\infty \frac{e^{-y}}{y} dy \tag{3}$$

where S is the storage coefficient of the aquifer, t is the time since the beginning of pumping, and y is the variable of integration. Note that the well function in Equation (3) can also be expressed with the following series (Kresic, 1997):

$$W(u) = ln\frac{0.5615}{u} + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{u^n}{n \cdot n!} \tag{4}$$

In the proposed approach, the value of $W(u)$ is calculated by means of Equation (4) by taking the upper limit of n is 100.

2.2 Hantush Model

Unsteady groundwater flow toward a fully penetrating well in a leaky-confined aquifer without aquitard storage is given by the Hantush model which is expressed as follows (Hantush and Jacob, 1955):

$$s(r,t) = \frac{Q}{4\pi T} W(u, r/B) \tag{5}$$

where B is the leakage factor, and $W(u,r/B)$ is the well function for the leaky confined aquifer whose value is the function of u and r/B. Definition of $W(u,r/B)$ and B are given as follows:

$$W(u,r/B) = \int_u^\infty \frac{e^{\left(-y-\frac{(r/B)^2}{4y}\right)}}{y} dy \tag{6}$$

$$B = \sqrt{\frac{Tb'}{K'}} \tag{7}$$

where b' and K' are the thickness and hydraulic conductivity of the confining bed (aquitard). Note that $W(u,r/B)$ values are tabulated for different u and r/B values by numerically integrating Equation (6) on MATLAB before executing the HS-Solver approach. In the optimization process, values of $W(u,r/B)$ are determined by conducting a bilinear interpolation between these tabulated values for the calculated u and r/B values.

2.3 Hybrid HS-Solver Optimization Approach

HS–Solver, which is proposed by Ayvaz et al. (2009), is a recently proposed hybrid optimization approach which integrates the global exploration feature of the HS and strong local search capability of GRG approach in the spreadsheet Solver add-in. Note that HS, proposed by Geem et al. (2001), is a heuristic algorithm which is inspired from the musical improvisation process. In musical processes, a pleasing harmony can be obtained by following three rules: i) playing a note randomly, ii) playing a note from harmony memory, iii) playing a note which is close to another one stored in harmony memory. Geem et al (2001) first adapted these musical rules to solve engineering optimization problems as: i) new decision variable values are selected randomly from the possible range, ii) new decision variable values are selected from harmony memory, iii) new decision variable values are further replaced with other ones which are close to their current values. Combinations of these rules in an optimization framework allow obtaining a global optimum solution in HS optimization algorithm.

Nowadays, electronic spreadsheets have become necessary tools to perform various engineering computations. Almost all the commercially available spreadsheet products such as Excel® include a built-in Solver add-in for solving unconstrained and constrained optimization problems by means of the GRG approach (Frontline Systems, 2018). Since Solver works on Excel®, all its computational features are accessible from the Visual Basic for Applications (VBA) platform. Therefore, HS and Solver processes have been hybridized on VBA platform by developing three separate VBA modules. The first module is for the stand alone HS and can be directly used to solve any optimization problem. The second module is developed for calling the Solver, which is created by using the macro recording feature of Excel®. The last module is used to integrate the HS and Solver modules to generate a hybrid optimization approach. Note that there are two options to integrate the HS and Solver processes in the last module. In the first option, the entire search space is explored by HS, then Solver gets the best result of HS as a starting point to precisely find the global optimum solution. In the second option, both HS and Solver run simultaneously such that all the solutions of HS are subjected to local search by Solver based on a small probability of P_c (Ayvaz and Geem, 2018). In the proposed approach, the second option is considered by setting the probability of $P_c = 0.10$ for all the solutions. The required solution parameters of HS-Solver are the harmony memory size (HMS), harmony memory considering rate ($HMCR$), pitch adjusting rate (PAR), fret width (*fw*), and the probability of P_c. A detailed description of these parameters and HS can be found in Geem et al. (2001) and Ayvaz et al. (2009).

2.4 Problem Formulation

The aquifer parameters can be identified by integrating HS-Solver approach with Theis and Hantush models for the confined and leaky-confined aquifers, respectively. In this integration, HS-Solver approach generates the associated aquifer parameters and these parameters are used in the Theis and Hantush models to estimate the drawdown of $s(r,t)$ in Equations (1) and (5), respectively. After this process, value of the objective function is calculated by means of the estimated and observed drawdowns at given distance and times. Note that the objective function used in this study is the sum of square errors (SSE) which is given as follows:

$$SSE(r) = \sum_{t=1}^{n_t}\left(s(r,t) - \tilde{s}(r,t)\right)^2 \tag{8}$$

where $SSE(r)$ is the calculated SSE value at a distance r, n_t is the number of monitoring time steps, and $\tilde{s}(r,t)$ is the observed drawdown at a distance r and at a time t which is obtained from the results of pumping test. After calculation objective function value, the parameter values of the aquifer system are adjusted by HS-Solver approach in order to minimize the calculated $SSE(r)$ value. Note that decision variables of the optimization model are T and S for the confined and T, S, and B for the leaky-confined aquifers.

3 Numerical Applications

The performance of the proposed hybrid approach is evaluated by considering two pumping test data given in Kresic (1997). The first test is conducted on a confined aquifer while the second one is conducted on a leaky-confined aquifer. For both tests, the identified aquifer parameters are compared with the ones which are determined by using the curve matching approach in Kresic (1997).

Furthermore, both problems have been solved 2 times for the same initial solutions in order to evaluate the model performance for HS and HS-Solver approaches. The HS-Solver solution parameters are taken as: $HMS = 10$, $HMCR = 0.95$, $PAR = 0.50$, $P_c = 0.10$, and $fw = (x_j^{max} - x_j^{min})/300$ where x_j^{max} and x_j^{min} are the maximum and minimum limits of the j^{th} decision variable.

3.1 Example 1: Confined aquifer

Figure 1 shows the confined aquifer system under consideration. A 24 hour pumping test is conducted to determine the T and S. The pumping rate of Q is kept constant at 0.008 m^3/s throughout the test and the corresponding drawdowns at the piezometers of P1, P2, and P3 are recorded. By assuming $r_1 = 5.5$ m, $r_2 = 40.5$ m, and $r_3 = 118$ m in Figure 1, the problem is solved by using both HS and HS-Solver based solution approaches. Figure 2 shows the convergence plots for both HS and HS-Solver approaches for each piezometer.

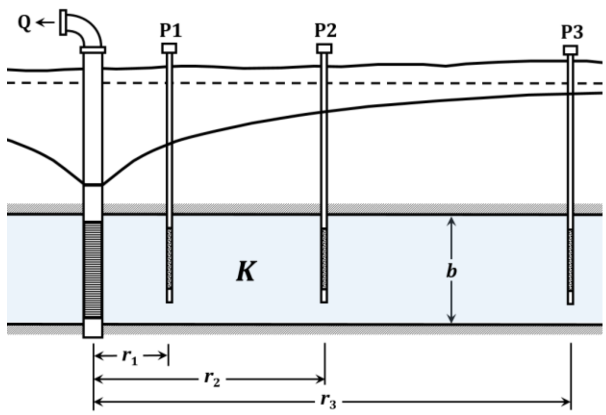

Figure 1: Cross-section of the confined aquifer under consideration

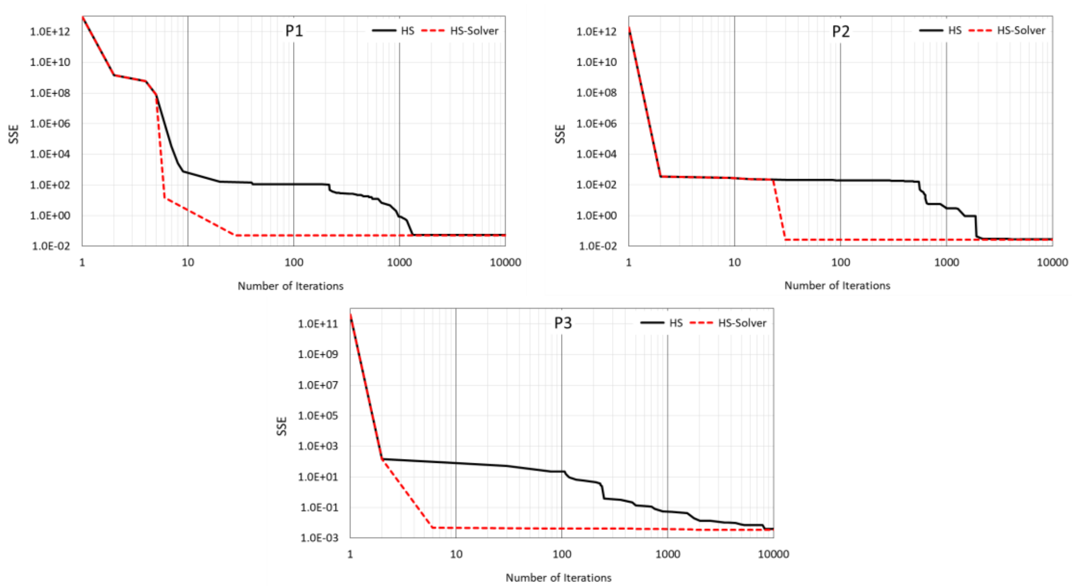

Figure 2: Convergence plots of the HS and HS-Solver approaches for (a): P1; (b): P2; (c): P3

As can be seen from Figure 2, both HS and HS-Solver start the search process from the same initial solutions since the same random numbers seeds are used. Although both approaches have similar trend in the earlier stages of the optimization process, it is observed a significant improvement in the objective function value when the Solver is included to the HS solution. For these outcomes, Table 1 and 2 compare the model results with those obtained by using the manual curve matching approach. Table 1 states that although the identified T and S values well agree with the ones obtained from the curve matching approach for P2 and P3, there is a small difference in both T and S for P1. The reason of this difference is associated with the distance between pumping well and P1. Since P1 is the closest piezometer to the well location, its corresponding drawdowns are greater than those obtained in P2 and P3. Therefore, the recorded drawdown data of P1 are much less curved than for P2 and P3 and matching the theoretical Theis curve is more arbitrary as expected (Kresic, 1997). This outcome is also observed in Table 2 such that the calculated SSE for P1 in the curve matching approach is approximately 10 times bigger than both HS and HS-Solver approaches. Note that Table 2 also includes a comparison of HS and HS-Solver in terms of the number of required iterations. As can be seen, for each piezometer, HS-Solver finds better SSE values by performing significantly fewer iterations than HS.

Solution approach	P1		P2		P3	
	T ($\mathrm{m^2/day}$)	S ($\times 10^{-5}$)	T ($\mathrm{m^2/day}$)	S ($\times 10^{-5}$)	T ($\mathrm{m^2/day}$)	S ($\times 10^{-5}$)
Curve matching	119.23	8.70	130.46	4.70	129.60	5.20
HS	127.87	5.69	129.60	4.81	129.60	5.10
HS-Solver	128.74	5.46	128.74	4.92	131.33	4.96

Table 1: Comparison of the estimated T and S values for each piezometer

Solution approach	SSE			Number of HS (Solver) Iterations		
	P1	P2	P3	P1	P2	P3
Curve matching	0.5497*	0.0306*	0.0056*	N/A	N/A	N/A
HS	0.0529	0.0271	0.0039	1494	6378	9196
HS-Solver	0.0519	0.0264	0.0035	120 (83)	130 (45)	1858 (38)

* This value is calculated by writing the results of Kresic (1997) into the Theis model.

Table 2: Comparison of the final SSE values and number of required iterations (the values in the parentheses correspond to number of Solver iterations)

3.2 Example 2: Leaky-confined aquifer

Figure 3 shows the leaky-confined aquifer considered in this example. A pumping test is conducted at a fully penetrating well in a confined aquifer. The aquifer is overlain by an aquitard which is overlain by an unconfined aquifer. The test is conducted by considering a constant Q of $0.012\ \mathrm{m^3/s}$ and the corresponding drawdowns are recorded at a piezometer which is $r = 128$ m away from the pumping well. By using these recorded data, the objective of the HS-Solver approach is to find T, S, and B for the given aquifer system. For this analysis, Figure 4 shows the convergence plot of the HS and HS-Solver approaches. As can be seen, similarly, both HS and HS-Solver start the search process with the same initial solution and proceed together in early iterations. After Solver gets the initial solution provided by HS, the calculated SSE value is significantly improved without requiring much more iterations. For these outcomes, identification results are compared in Table 3 in terms of the aquifer parameters, SSE values, and number of required iterations.

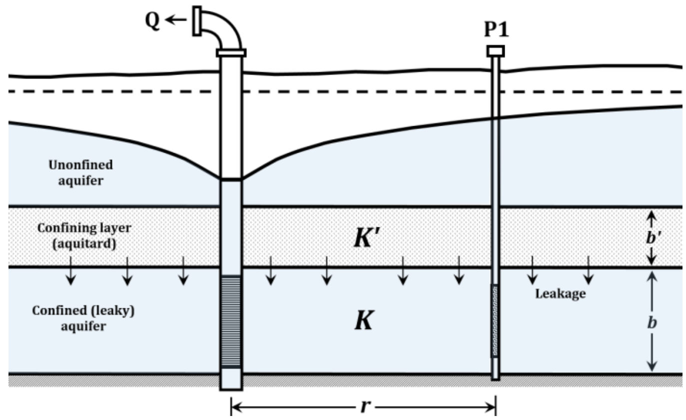

Figure 3: Cross-section of the leaky-confined aquifer under consideration

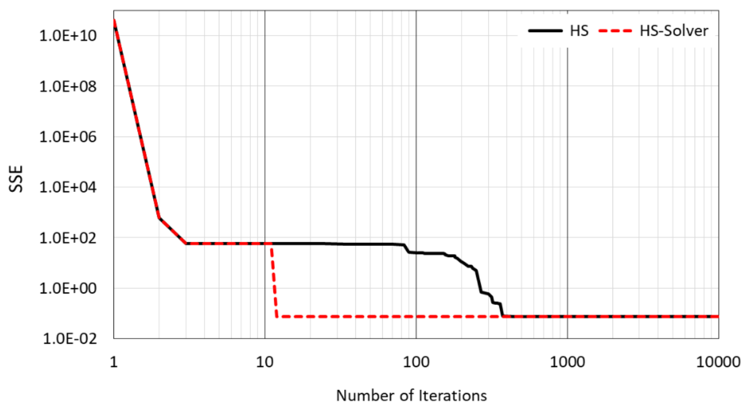

Figure 4: Convergence plots of the HS and HS-Solver approaches

Solution approach	T (m^2/day)	S ($\times 10^{-5}$)	B (m)	SSE	Number of HS (Solver) Iterations
Curve matching	257.47	1.09	640.00	3.3678*	N/A
HS	160.48	1.25	460.03	0.0754	9076
HS-Solver	159.93	1.26	457.90	0.0753	1053 (129)

* This value is calculated by writing the results of Kresic (1997) into the Hantush model.

Table 3: Comparison of the estimated T, S, and B values and final SSE values and number of required iterations (the values in the parentheses correspond to number of Solver iterations)

It can be seen from Table 3 that identified T, S, and B values of HS and HS-Solver well agree with each other although there are some differences from the ones obtained by using the manual curve matching approach. This difference is associated with the inaccuracy of the manual curve matching approach such that the final SSE value of the curve matching approach is approximately 45 times bigger than both HS and HS-Solver approaches. When the number of required iterations are compared, it can be seen that HS-Solver required 1053 HS and 129 Solver iterations while approximately the same result is obtained by using HS in 9076 HS iterations.

4 Conclusions

In this study, the hybrid HS-Solver approach is first applied to the solution of aquifer parameter estimation problems by using the results of pumping tests. In the proposed approach, drawdown values at given distance and monitoring times are calculated by means of Theis and Hantush models for the confined and leaky-confined aquifers, respectively. These models are then used in conjunction with HS-Solver approach to determine the given aquifer parameters. Evaluation of the proposed approach is conducted by solving two pumping test examples. Identified results indicate that use of a simulation-optimization approach outperforms the inadequacy of the manual curve matching approach. Furthermore, use of a local search approach in a heuristic optimization approach significantly reduces the number of required iterations to find an optimum solution.

References

Ayvaz, M.T., Geem, Z.W. (2018). Optimum design of the booster chlorination systems by using hybrid HS-Solver optimization technique, *PAJES* (doi: 10.5505/pajes.2017.22587).

Ayvaz, M.T., Kayhan, A.H., Ceylan, H., Gurarslan, G. (2009). Hybridizing the harmony search algorithm with a spreadsheet 'Solver' for solving continuous engineering optimization problems, *Engineering Optimization*, Vol. 41, pp. 1119-1144.

Frontline Systems (2018). Premium Solver's web site. *http://www.solver.com/premium-solver-platform#tab2*

Geem, Z.W., Kim, J.H., Loganathan, G.V. (2001). A new heuristic optimization algorithm: Harmony Search, *Simulation*, Vol. 76, pp. 60-68.

Hantush, M.S., Jacob, C.E. (1955). Non-steady radial flow in an infinite leaky aquifer. *Trans., Am. Geophys. Union*, Vol. 36, pp. 95-100.

Huang, Y.C., Yeh, H.D., Lin, Y.C. (2008). A computer method based on simulated annealing to identify aquifer parameters using pumping-test data, *Int. J. Numer. Anal. Meth. Geomech*. Vol. 32, pp. 235–249.

Kresic, N. (1997). *Quantitative solutions in hydrogeology and groundwater modelling*, CRC Press, New York.

Michalewicz, Z. (1992). *Genetic algorithm + data structure = evolution programs*, Springer–Verlag, NewYork.

Sahin, A.U. (2018). A particle swarm optimization assessment for the determination of non-darcian flow parameters in a confined aquifer, *Water Resources Management*, Vol. 32, pp. 751-767.

Samuel, M.P., Jha, M.K. (2003). Estimation of aquifer parameters from pumping test data by genetic algorithm optimization technique, *Journal of Irrigation and Drainage Engineering*, Vol. 129.

Shannon, M.W. (1998). *Evolutionary algorithms with local search for combinatorial optimization*. Thesis (PhD), University of California, USA.

Sun, N.Z. (1994). *Inverse problems in groundwater modeling*, Kluwer Academic, Dordrecht, the Netherlands.

Theis, C.V. (1935). The relation between the lowering of the piezometric surface and the rate and duration of discharge of well using groundwater storage, *Trans., Am. Geophysi. Union*, Vol. 16, pp. 519–524.

Willis, R., Yeh, W.W.G. (1987). *Groundwater system planning and management*, Prentice-Hall, Englewood Cliffs, N.J.